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Factors and dimensions in legal reasoning

Most analyses of legal reasoning have focused on the application of rules, or on teleological (goal or value-based) reasoning, or on dialectical interactions. Here we shall focus on a different aspect of reasoning, which, though often neglected, represents an important aspect of legal problem-solving and argumentation. This consists in referring to what we shall call factors, or factor/outcome-links. After characterising the notion of a factor, distinguishing in particular between binary factors and scalable ones (dimension) we shall provide an account of the use of factors in legal reasoning.

1 Factors

Recognising a factor consists in assuming that certain circumstances prompt our physical or mental behaviour in a certain direction. The recognition of a factor needs to be distinguished from the adoption of a rule. When one adopts a rule, one is committed to derive the conclusion of the rule (if its conditions are satisfied), unless defeating circumstances occur. On the contrary, when one recognises a factor, one is only committed to take it into consideration, according to its relevance, whenever it obtains. Consider for instance the difference between recognising that the educational nature of a certain use of intellectual property is a factor favouring the conclusion that this use is allowed (it is a fair use), and endorsing a rule to the effect that every form of educational use is allowed.

Factors, as they need to be distinguished from rules, need also to be distinguished from goals or values. A factor promoting a certain outcome is no goal to be achieved (nor a value to be implemented) in the future: It is rather a feature of the pre-existing context that favours a certain outcome (a certain conclusion in legal reasoning and decision-making). For example, the fact that a literary work has a factual, rather than fictional, nature is a factor favouring the conclusion that the use of that work is allowed (it is a fair use), but factuality is not a goal (a value) we want to achieve: Why should we promote factual rather than fictional works? Similarly, a crime having been committed with cruelty is a factor for increasing the punishment of its author, but cruelty in crimes is not a goal we want to achieve as much as possible.

1.1 Binary Factors and Dimensions

Some factors are binary. A binary factor either is fully present in a case (and the factor's outcome is favoured) or is fully absent: It does not make sense (or it is anyway irrelevant) to view the binary factor as being present to a higher or lower degree. For instance, either one is a woman (a man) or one is not, and being a woman (a man) may be viewed (by those who see gender balance as a valuable goal) as a factor with favours one's appointment in an area where women (men) are underrepresented.

Some other factors appear to be scalable: The more intensely they are present, the more they favour a certain action. For example, the malice of the author of a crime is a factor that increasingly favours his or her punishment: The more malice one has exhibited in committing a crime, the more punishment is required.

We shall call a scalable factor a dimensional factor, or more simply, a dimension, using the terminology proposed by K. Ashley and E. Rissland ([Ashley, 1990]; [Rissland and Ashley, 1987]), who introduced the use of dimensions in the analysis of legal cases. Some dimensions can be viewed as having a double direction: Up to a certain degree they favour a certain outcome, above that degree they favour the opposite outcome. For instance, the goodness of the motives for which a crime has been committed provides a ground for diminishing punishment (and to diminish it more, the better being the motives), while the negative goodness (the badness) of such motives provides a ground for augmenting punishment (and for increasing it more, the worse being the motives): We may thus see the quality of the motives of a crime as being a continuous dimension (ranging from the noblest and worthiest motives, to the most abject and vile ones), having a tendency to influence the amount of the penalty towards a progressive increase.

Transforming a continuous dimension into a binary factor (either motives are good or are bad, and there is a corresponding fixed increase or decrease of the sanction), is a strategy of bounded rationality for simplifying one's decisions, for making them easier and more foreseeable, though at the price of a reduced capacity of apportioning legal conclusions to the specific features of individual cases.
1.2 Factors and Principles

Factor-based reasoning plays a central role in moral and legal reasoning, though it is rarely specifically discussed under this heading\(^2\), being rather approached in connection with the idea of a principle. For instance, the two paradigmatic examples of principles that are to be found in [Dworkin, 1977], a contribution that originated a vast debate on the notion of a principle, seem to be properly classifiable as factor/outcome-links. The first principle is taken from case *Riggs vs Palmer*, a decision of a New York court (in 1889) that denied the inheritance to a person named in the will of his grandfather, who had killed the grandfather in order to inherit from him. The ground of this decision is idea that:

No one shall be permitted to profit by his own fraud, or to take advantage of his own wrong, or to found any claim upon his own iniquity, or to acquire property by his own crime.

The second principle is taken from case *Henningsen vs Bloomfield Motors Inc.*, a decision from a New Jersey's Court (in 1960). In this case the judges recognised the liability of the manufacturer of a faulty car for the damages suffered by the buyer, though the contract between the manufacturer and the buyer included a limitation of liability. Their conclusion is supported by the idea that:

The courts generally refuse to lend themselves to the enforcement of a bargain in which one party has unjustly taken advantage of the economic necessities of the other.

Both principles can easily be rephrased into factor/outcome-links: The first says that the fact than one has obtained a profit (advantage or property) through fraud (wrong or iniquity) favours the conclusion of not allowing one to keep that profit; the second says that the fact that one party in a contract has unjustly taken advantage of the economic necessities of the other party favours the conclusion that the contract should not enforced.

This interpretation is confirmed by Dworkin's description of the way in which principles work:

[A principle] states a reason that argues in one direction, but does not necessitate a particular decision. If a man is or is about to receive something, as a direct result of something illegal he did to get it, then that is a reason which the law will take into account in deciding whether he should keep it. There may be other principles or policies arguing in the other direction—the policy of securing title, for example, or a principle limiting punishment to what the legislature has stipulated. If so, our principle may not prevail, but this does not mean that it is not a principle of our legal system, because in the next case, when these competing considerations are absent or less weighty, the principle may be decisive. [Dworkin, 1977, 26]

Also other examples of principles indicated by Dworkin could be rephrased as factor links: An act being formally regular favours the conclusion that the act is enforceable, a punishment not having been explicitly established by the legislator favours the conclusion that the courts should not order that punishment, and so on.

To express the connection between a factor and the outcome it favours, one may use the word *reason*, saying that the factor is a reason for its outcome. However, we should pay attention that in this connection the word "reason" may refer to different objects and play different functions: In particular, when by speaking of a reason for a certain conclusion, sometimes we mean the ground directly supporting that conclusion, sometimes we mean what support that conclusion only indirectly, through additional reasoning steps. For instance, we may draw the different distinction:

- The factor [one has obtained a profit though fraud], assuming the factor-link [having obtained a profit though fraud favours not being allowed to keep the profit], is a reason directly for concluding that one is not allowed to keep the profit (though it may insufficient to get to that conclusion, unless other factors are present).
- Any goal or value which is going to be promoted through the recognition of this factor/outcome-link (for example, the goal of reducing frauds, or of enhancing mutual trust) provides a reason (a rationale) for endorsing the factor-link, and thus only indirectly support the conclusion that can be obtained thanks to that link (the conclusion that one is not allowed to keep the profit).

It is particularly important to distinguish the notion of a factor/outcome-link from the notions of a value, as both values and factors are frequently merged under the heading of principles.\(^3\)
On the contrary, we believe that the cognitive role of values is different from that of factors. Values (together with goals) find their proper role within teleological reasoning (which may support the adoption of both rules and factor-links), while factor-based reasoning provides a backward looking alternative to forward-looking teleological reasoning.

Similarly, we need to distinguish factors and defeasible rules, which (or at least some of which) also are frequently called "principles." While a factor (combined with the factor/outcome-link) only is a contributory reason for the favoured result, the antecedent of a defeasible rule (combined with the rule itself) is a defeasibly sufficient reason for the rule's effect. In other words, when believing that a certain factor exists in the current situation, I am not committed to assume that, in the absence of reasons to the contrary, the favoured result obtains. On the contrary, when believing that the antecedent of a defeasible rule is satisfied, I am committed to assume that in the absence of reasons to the contrary, the rule's consequent obtains.

We shall not provide a precise characterisation of the notion of a principle. It seems to us that it is better to use term principle in a generic way, namely, as expression or the importance of a certain legal content (a piece of normative information), and of the fact that this content is the premise from which further significant contents (like legal rules, instrumental legal values, specific factors) can be inferred. In fact, it seems to us that usually qualifying a legal content as a principle entails no commitment to a specific logical function or form, or to a specific origin. Thus, we can correctly speak of a principle with regard to all of the following: the statement of a legal value, like [human dignity is a fundamental legal value, to be protected and advanced]; an indefeasible rule, like [nobody shall ever be tortured]; a defeasible rule, like [workers have a right to strike]; a factor/outcome-link, like [the fact that an interpretation corresponds to the textual meaning of a statute favours its endorsement].

As we can use the term principle to denote legal contents having different logical structures, we can use it to refer to contents having different origins. In particular, we can correctly speak of a principle non only with regard to ideas pertaining to legal tradition or political morality, but also with regard to contents that are expressed in positive sources of the law: in a constitution (like the principle that workers have a right to strike, included in the Italian Constitution), in ordinary legislation (like the principle of vicarious liability of employers, stated in the Italian Civil code), in case law (like the principle that health damages are to be compensated, introduced by Italian judges some years ago).

1.3 Factors, Dimensions and Standards

The notion of a factor, and in particular of a dimension, can be connected to a further kind of legal content, legal standards, that is, those legal models of behaviour which, according to [Pound, 1954], have the following characteristics:

1. They all involve a certain moral judgment upon conduct. It is to be "fair," or "conscientious," or "reasonable," or "prudent," or "diligent." (2) They do not call for exact legal knowledge exactly applied, but for common sense about common things or trained intuition about things outside of everyone's experience. (3) They are not formulated absolutely and given an exact content, either by legislation or by judicial decision, but are relative to times and places and circumstances and are to be applied with reference to the facts of the case in hand.

From our perspective, a standard (fairness, good faith, reasonableness, care) appears to be a dimension, and in particular a property that increasingly favours a positive evaluation of the activities to which it applies (the more one is fair, conscientious, reasonable, prudent, the more praiseworthy one's behaviour is).

However, the application of the standard involves also other kinds of legal information. The standard is usually combined with one or more rules, legally requiring that a certain level of the standard is maintained in certain domains (for instance different rules of law may require different levels of care in different activities and with regard to different professions).

These rules can be cast as obligatory rules: [Producers have a duty of care to their customers], [Contractual parties must be fair toward one another], and so forth.

Determining the required dimensional levels may require teleological considerations: Given the goals of facilitating the formation of contracts, promoting mutual trust, and preventing litigation, but also of ensuring the liberty of contractors and limiting the cost of contracting, what minimum level or fairness and consciousness should be legally required from the parties? The required level of the standard can also be determined with reference to an exemplar or prototype: the normal person, the good father of a family (bonus pater familias), the good medical doctor, lawyer, accountant, and so on.

Establishing whether this level has been achieved requires extralegal knowledge, proper to the domain of activity that is being considered. For instance, to establish whether a doctor has behaved with the required level of medical care, one needs to consider what medical information was accessible to him or her, whether this information was used correctly understood and applied in his or her work, whether a sufficient degree of attention was maintained, and so on.
1.4 Factors in Legislation

In his famous characterisation of principles, [Dworkin, 1977] identifies further aspects of them (besides the fact that they operate as factors): Principles are dependent on morality (rather than on expediency); they have the function of protecting individual rights (rather than advancing social goals); they are included in legal culture (rather than having been expressly stated by a legislator). Not all such characters apply to all factors. In particular, the relevance of certain factors may depend upon legislative choice: The legislator may state explicitly what factors are, or are not, to be considered when taking a particular decision.

For example, the Italian criminal code specifies what factors may lead to an aggravation of a crime (like the fact that it was committed with cruelty), and what factors may lead to its attenuation (like the circumstance that the author of the crime acted in a state of wrath, caused by the behaviour of the victim), and requires the judge to balance those factors to establish whether the aggravation or the attenuation prevail, and consequently increase or decrease punishment. As another example of legislative factors, consider for example the US Copyright Act, which mandates, at section 107, that:

In determining whether the use made of a work in any particular case is a fair use the factors to be considered shall include:

- the purpose and character of the use, including whether such use is of a commercial nature or is for non-profit educational purposes;
- the nature of the copyrighted work;
- the amount and substantiality of the portion used in relation to the copyrighted work as a whole, and
- the effect of the use upon the potential market for or value of the work.

The factors in section 107 are bi-directional dimensions:

- the more the use is no-profit the more fair use is favoured, the less it is so the more copyright protection is favoured;
- the more the content is factual the more fair use is favoured, the less the content is factual (the more the content is fictional) the more copyright is favoured;
- the smaller the used portion the more fair use is favoured, the bigger the portion the more copyright is favoured;
- the smaller the impact upon the market value of the work, the more fair use is favoured, the larger the impact the more copyright is favoured.

This example clarifies the difference between factors on the one hand and values on the other hand. Factors are not results one should aim at achieving as much as possible in the future (through a specific action or through the general practice of a rule). They are rather features of the existing (or pre-existing) situation favouring a certain normative conclusion: While values are forward-looking, factors are backward-looking.

The relationship between factors and values is not identity but teleological inference: The reason why a factor is recognised as promoting a certain outcome consists in the fact that through recognising this factor/outcome connection (by giving the factor a certain weight when taking a decision) one promotes certain values.

For example, by giving relevance to the factual nature of a work as favouring its fair use, one promotes the values of knowledge and information, while by giving relevance to its fictional nature as a factor for copyright protection one promotes creativity.

However, when one reasons with factors, one does not need to refer to the connection between factors and the underlying values, and one does not even need to be aware of such connections. This simplifies the reasoning of the agent, who can find "satisficing" solutions (as Herbert Simon would say) without engaging in the complexities of teleological reasoning. One needs to move up to values only when factor-based reasoning leads to absurd or meaningless outcomes, or when one has to adjudicate a conflict of competing factors. Under such circumstances, one needs to engage in rationalisation, trying to secure the doubtful or contested factor/outcome-links by anchoring them to values.

A failure to provide this anchorage signals that a factor/outcome-link may have to be abandoned, since it does not play any acceptable function. For example, up to a certain time, in Italian criminal law, the fact of acting out of sense of honour was considered to be a factor favouring a considerable reduction in criminal punishment (especially in cases of homicide between partners or relatives, in particular when extramarital relations were at issue). However, at a certain time this factor started to have less and less importance, as people started to look in a different way at family relationships.
1.5 Factors in Case-Based Reasoning

Factor-based thinking is an essential aspect of case law. For a clear reference to factors and factor-based reasoning, we shall consider the case Playboy Enterprises Inc. vs George Frena (1993). In this case the judges had to decide whether the publication of some Playboy photos on a subscription-based bulletin board, maintained by George Frena, implied a infringement under 15 U.S.C. Section 1114. One of the central points to be established concerned the likelihood that confusion was caused by the fact that Frena used the word "Playboy," a registered mark, in presenting those photos. Here is how the judges describe how this evaluation needs to be performed:

The following factors are highly relevant in deciding whether there is a likelihood of confusion: "(1) the type of mark at issue; (2) similarity of marks; (3) similarity of product or services; (4) identity of purchasers and similarity of retail outlets; [...] (6) the defendant's intent; and (7) actual confusion." Ice Cold Auto Air, 828 F.Supp. at 935 (citing Freedom Sav. and Loan Ass'n, 757 F.2d at 1182-83). The Court, however, is not required to specifically mention each of these factors in making its decision. See Univ. of Georgia Athletic Ass'n v. Laite, 756 F.2d 1535, 1542 (11th Cir.1985) (analyzing the factors in the context of a claim of unfair competition). Rather than simply determining whether a majority of these factors indicate a likelihood of confusion, a court must "evaluate the weight to be accorded the individual factors and then make its ultimate decision." [...] An analysis of fewer than all seven factors may support a finding of likelihood of confusion. See Univ. of Georgia Athletic Ass'n, 756 F.2d at 1543. In the Eleventh Circuit, the type of mark and evidence of actual confusion are the most important factors.

Thus, according to the judges, the conclusion that there is a likelihood of confusion (a conclusion that may lead, in its turn, to establish that there was a trademark infringement) depends upon a number of scalable factors. These factors are to be weighed together, in their variable combinations, to see whether they are sufficient to support that conclusion in a particular case.

1.6 The Role of Factors in Practical Inference

To reason with factor we need to do all of the following:

- first collect factors favouring or disfavouring a certain outcome, both pros and cons,
- then compare the combined strength of the two sets of factors, and
- finally, according to this evaluation, adopt a determination (or refrain from forming it).

We may distinguish two kinds of such determinations. Firstly the determination may concern a specific legal conclusion for the case at hand. Secondly, the determination may concern a general commitment: One shall endorse the conclusion whenever that combination of factors is present. This means that this set of factors becomes the antecedent of a conditional rule, to which one gets committed. Assume, for example, that in a certain legal system, the educational nature of a certain use of a copyrighted work favours the conclusion that it is a fair use. A lawyer could argue that the value of education is so overwhelming that this factor/outcome-link should originate a conditional rule: The law should be viewed as including a rule to the effect that any educational use is fair. The opposite passage can also take place: one may transform a conditional instruction into a factor/outcome-link. Rather than rigidly committing oneself to that instruction, one may choose to view the instruction's antecedent as a factor which needs to be balanced in the concrete case with factors to the contrary. For example, one may argue that under the conditions of the Internet society, where circulation of information has become ubiquitous and costless (so that one cannot limit the unwanted uses of one's work, once it has been made accessible), a rule to the effect any educational use is fair would involve an inadmissible compression of the rights of the authors: Rather than as a rule, the connection between educational and fair use it to be viewed as a factor/outcome-link, the relevance of which has to be evaluated case by case, according to the ways and circumstances of the educational use.
1.7 Factors, Dimensions, and Prototypes

It has been frequently observed that many legal concepts are not characterised through a set of necessary and jointly sufficient conditions, but rather through a prototype, or through a set of prototypes, connected by family resemblances. Prototypical meanings are not limited to legal language: It seems that the concept we are most familiar with (the notions of a table, a plate, a chair, a fruit, a game) are not understood through precise definitions, but rather through prototypes or exemplars. These concepts elicit a set of typical cases to which we compare the objects of our experience: We conclude that a concept applies to an object when the similarities between the concept's prototypes and the object outweigh their differences (and no alternative concepts gives a better match).

In general, to check whether a certain entity is an instance of a prototypical notion one has to consider: (a) to what degree and in what combination the features characterising the prototype are present in that entity, and (b) whether the entity has additional features, which are absent in the prototype, and which would hinder the present prototypical features from playing their normal function.

A formal model of prototypical reasoning has been proposed by [McCarty, 1982], but before that some legal theorists, especially in the German tradition, have dedicated much effort to the idea of a legal prototype, assuming that a Typus is "characterised by features that do not need always to be present together, or that can be present to different extents in the concrete instances" ([Larenz, 1992,131]; my translation). As examples of prototypically characterised notions, [Larenz, 1992] indicates the notion of the "essential component of an object" (wesentliche Bestandteil einer Sache), which depends on the measure in which the component contributes to the normal functions of the object, or the notion of the guardian of an animal (Tierhalter), a qualification which is dependent on the extent to which one has control over the animal in one's own interest. He also argues that for establishing whether a certain entity is an instance of a Typus one has to refer to values, that is, one has to consider whether-with regard to the relevant values-it would be appropriate that the entity produces the legal effects that are connected to the Typus (on Typen, see also [Kaufmann, 1965], and [Hassemer, 1968]).

Though the notions of a factor and a dimension do not exhaust the idea of a prototype, they can provide an insight into prototypical thinking.

Let us assume that the variable and exchangeable features that characterise legal prototypes are viewed as factors and dimensions. Correspondingly, we characterise a prototypical qualification as being favoured by certain factors or along certain dimensions, and we view prototypes as consisting in the optimal combinations of such factors and dimensions. For example, assume that the law grants particular warranties to employees (protection against unjustified dismissals, against mobbing and discrimination, health insurance, and so on), warranties that are not available, or not to the same extent, to independent workers. Assume also that the qualification of a worker as an employee would be favoured to the extent that the worker is dedicating a larger proportion of his working time to one work-giver, is following the directions of the work-giver, is working within the premises of the work-giver or using the work-giver's tools.

With regard to such dimensions, not only can we build the notion of a prototypical employee (a person who is working full time for a single employer, under detailed directions, within the employer's premises and using the employers' tools), but we get a multi-dimensional space were we can locate different real or hypothetical cases. For instance, assume that we locate within this dimensional space case c₁, where a judge concluded for the existence of an employment relationship between a₁ and b₁, where a₁ is a woman who was working in her house, but was dedicating 80% of her working time to the work-giver b₁, and following b₁'s detailed instruction, and mostly using b₁'s tools.

Once we have characterised the dimensional space, and have located within this space the known positive or negative examples at our disposal, we can reason analogically, in order to establish whether a certain hypothetical or real case can be characterised as an instance of the prototypical concept. This would be done by considering what factors and dimensions apply to the new case, and comparing the new case to the known instances of the prototypical notion (according to a fortiori reasoning or other ways of analogising).

For instance, consider case c₂ where a₂ is a man dedicating 90% of his time to work-giver b₂, and a₂ is following b₂'s directions and using b₂'s tools to the same extent in which a₁ was following b₁'s directions, used b₁'s tools, and so on. Under these circumstances, a fortiori reasoning allows us to conclude that indeed also the relation between a₂ and b₂ is an employment relationship. Consequently, we can conclude that also b₂, like b₁, has rights to health insurance, to a certain degree of stability in his job, and so on.

When a fortiori reasoning cannot be used (or when one wants to challenge past classifications), one needs to resort to values. The assignment to a prototype involves reference to values since values explain why factors and dimensions promote the classification under a certain prototype: Such classification determines certain legal effects, and connecting these legal effect to the relevant factors and dimensions (by means of the intermediate prototypical concept) contributes to some values.

For instance, when we cannot or do not want to rely on analogies from past exemplars of relationships between workers and work-givers, which have already been qualified as instantiating or not instantiating the employment relationship, we need to
engage in teleological reasoning: We need to examine to what extent the existence of an employment relation under certain conditions (when certain factors are present) would contribute to realise the values that underlie a stable employment relationship (mutual trust, security, freedom from arbitrary power, the chance to develop long term life projects, and so on), and weigh these values against the values that may be satisfied by a more flexible arrangements (economic freedom, efficiency, increased access to work, and so forth).

This evaluation may lead us to restructure the dimensional space, adding certain exemplars or even changing the qualifications of previously classified exemplars (for example, seeing instances of employment relationships where we previously saw independent work, or vice versa).

Finally, the possibility of passing from factors and dimensions to rules, and vice versa, through theory-construction processes, explains the dialectical movement of legal thinking between the prototypical characterisation of legal notions and the attempt to capture these notions through precise definitions.

2 Reasoning with Factors

Factors pointing to opposite directions need to be compared and evaluated, in order to establish what outcome is indicated by their combination.

One possible approach consists in relying on teleological rationalisation, that is, in ascending to the values that are promoted or impaired by recognising certain factors or certain combinations of them. Then we can assume that the comparison of the factors reflects the relative importance of the corresponding values.

However, one should not rely too much on teleological reasoning, which should be used for rationalising and fixing the outcome of intuitive evaluations, rather than as an autonomous source of practical determinations. It is more promising to appeal to precedents, and transfer to new cases the evaluations that were made in the past. First we shall introduce an example and examine some plausible reasoning moves, and then we shall try to provide a theory that synthesises these moves. Finally, we shall extend our model with reference dimensions (scalable factors).

2.1 An Example in Factor-Based Reasoning

The following example addresses the issue of whether one's stay in another country changes one's fiscal domicile with respect to income tax (the example is adapted from [Prakken and Sartor, 1998]).

We indicate the direction of the factors with arrows. In general, we write $F^{↑}$, to indicate that factor $F$ favours outcome $ϕ$, while we write $F^{↓}$, to indicate that factor $F$ disfavours outcome $ϕ$. However, since in our example we are only concerned with one outcome (the change in fiscal domicile) we shall leave the outcome implicit: We write $F^{↑}$ to indicate that $F$ is a pro-change factor, and $F^{↓}$ to indicate that $F$ is a con-change factor.

Assume that the following pro- and con-change factors can be identified in legislation, doctrine, or precedents:

- pro-change is that the taxpayer's house was given up [gave up house↑], while con-change is that the house was kept [kept house↓];
- pro-change is that the taxpayer's company is based in the foreign country [foreign company↑], while con-change is that the company is based in the old country [domestic company↓];
- pro-change is that the duration of the stay abroad is long [long stay↑], while con-change is that the duration is short [short stay↓];
- pro-change is that one has minor assets [minor assets↑] in one's country, while con-change is the fact that one has large assets there [major assets↓].

We do not presume that each factor receives a definite value in each case: For example, the duration may be neither long nor short, so that it does not push the decision in any direction.

Assume that a binding precedent $Prec_n$, which was decided for change, is characterised by the following factors: the taxpayer had a long duration contract for working abroad, was working for a domestic company, and kept his domestic house (see Table 1).
2.2 Factor-Based A-Fortiori Reasoning

Consider now the new case, let us call it New\textsubscript{a1}, which is described in Table 2.

<table>
<thead>
<tr>
<th>Pro-change factors</th>
<th>Con-change factors</th>
<th>Decision</th>
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<tbody>
<tr>
<td>long stay↑</td>
<td>domestic company↓</td>
<td>?</td>
</tr>
<tr>
<td>minor assets↑</td>
<td>kept house↓</td>
<td></td>
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Table 2: Case New\textsubscript{a1}

The new case concerns a worker having a long-term contract for a domestic company, who kept his house and had small assets in his home country. We need to establish what we should conclude with regard to the new case New\textsubscript{a1}, if we want to be consistent with decision in Prec\textsubscript{a} (in Table 1).

Observe that the Prec\textsubscript{a} expresses two messages:

- The first message is that its pro-factor [long stay↑] is sufficient for having a pro-change decision (unless defeated by contrary factors). This follows from the fact that a change decision was taken in Prec\textsubscript{a}, where only this pro-change factor was present.
- The second message is that factor [long stay↑] outweighs the combination of con-factors [domestic company↓] and [kept house↓]. This follows from the fact that Prec\textsubscript{a} has decision change, though both these con-factors were present.

Let us see what is the relevance of these messages for New\textsubscript{a1}. This case shares with Prec\textsubscript{a} one pro-change factors, i.e., [long stay↑], and also includes Prec\textsubscript{a}'s con-factors [domestic company↓] and [kept house↓]. The only difference which emerges from the factor-based description of the two cases is the following: New\textsubscript{a1} has one additional pro-change factor, that is [minor assets↑].

It seems that a reasoner should conclude that also in New\textsubscript{a1} the fiscal domicile has changed. In fact, if in Prec\textsubscript{a} [domestic company↓] and [kept house↓] were outweighed by [long stay↑] alone, a fortiori they should be outweighed when [long stay↑] is joined by an additional pro-change factor, [minor assets↑].

Let us consider now another new case, New\textsubscript{a2}, which is described in Table 3.

<table>
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<tr>
<th>Pro-change factors</th>
<th>Con-change factors</th>
<th>Decision</th>
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<tbody>
<tr>
<td>kept house↓</td>
<td></td>
<td>?</td>
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Table 3: Case New\textsubscript{a2}

Assume that neither of [domestic company↓] or [foreign company↑] applies to New\textsubscript{a2}, since the taxpayer is a free-lance worker. Thus, New\textsubscript{a2} is characterised by only two factors, [long stay↑] and [kept house↓].

By comparing New\textsubscript{a2} and Prec\textsubscript{a}, we can see that New\textsubscript{a2} contains the same pro-factors as Prec\textsubscript{a}, i.e., [long stay↑] and only one of Prec\textsubscript{a}'s con-change factors, i.e., [kept house↓].

Therefore, we may conclude that the message of Prec\textsubscript{a} also applies to New\textsubscript{a2} and dictates the same outcome, i.e., change. In fact, if the pro-change factor [long stay↑] outweighed in Prec\textsubscript{a} the combination of the two con-factors [domestic company↓] and [kept house↓]; a fortiori it should outweigh in New\textsubscript{a2} just one of those con-factors.

2.3 Factor-Based Inference

Let us try to identify reasoning schemata that may licence the inferences we have just presented.
We can conclude that the facts of New$_{a1}$ lead to the same conclusions as Prec$_{a}$, on the basis of the following idea: When the factors favouring a conclusion $q$ prevail over the factors against $q$, then adding additional factors favouring $q$ (the winning conclusion) should not change the outcome of the comparison (but rather strengthen such outcome). This is the inference pattern we call *additive a fortiori*, and that we may describe as follows (we write $F^\uparrow q$ to denote a set of factors all of which favour a certain conclusion $q$, and $F^\downarrow q$ to denote a set of factors all of which disfavour conclusion $q$):

**Reasoning schema: Additive a fortiori**

- $F^\uparrow q$ outweighs $G^\downarrow q$; and
- $F^\uparrow q$ is at least as inclusive as $F^\uparrow q$ ($F^\uparrow q \subseteq F^\uparrow q$)

IS A DEFEASIBLE REASON FOR

- $F^\uparrow q$ outweighs $G^\downarrow q$

Note that by a set of factors $F$ being at least as inclusive than a set of factors $F'$, we mean that $F'$ is a subset of $F$ (which we express, using $\subseteq$ for set inclusion, as $F' \subseteq F$), namely that all factors in $F'$ are also contained in $F$.

On the basis of this inference, the reasoner believing that the only pro-factor of Prec$_{a}$ could outweigh the two con-factors of that case, will conclude that also the couple of pro-factors of New$_{a1}$ (including the pro-factor of Prec$_{a}$) should *a fortiori* outweigh the same couple of con-factors.

This will lead one to conclude that the inference one can make by referring to these pro-factors strictly defeats any inference one can make appealing to the con-factors.

There also is a different situation in which a precedent's outcome *a fortiori* dictates the decision of a new case: The new case rather than having more factors favouring the precedent's outcome, has fewer factors against that outcome: We can conclude that the facts of New$_{a2}$ also lead to the same outcome as Prec$_{a}$ according to the idea that when a set of factors favouring conclusion $q$ outweighs a set of factors against $q$, then the outcome should not be changed (but rather strengthened) by eliminating factors against the winning conclusion.

This is the inference pattern we call *subtractive a fortiori*, which we describe through the following reasoning schema:

**Reasoning schema: Subtractive a fortiori**

- $F^\uparrow q$ outweighs $G^\downarrow q$; and
- $G^\downarrow q$ is no more inclusive than $G^\downarrow q$ ($G^\downarrow q \subseteq G^\downarrow q$)

IS A DEFEASIBLE REASON FOR

- $F^\uparrow q$ outweighs $G^\downarrow q$

Note that by a set of factors $A$ being no more inclusive that a set of factors $B$, we mean that $A$ is a subset of $B$, i.e., that all elements (all factors) in $A$ are also contained in $B$.

The two inference patterns can be merged into reasoning schema *bidirectional a fortiori* which covers the two cases:

**Reasoning schema: Bidirectional a fortiori**

- $F^\uparrow q$ outweighs $G^\downarrow q$;
- $F^\uparrow q$ is at least as inclusive as $F^\uparrow q$; and
- $G^\downarrow q$ is no more inclusive than $G^\downarrow q$

IS A DEFEASIBLE REASON FOR

- $F^\uparrow q$ outweighs $G^\downarrow q$

*A fortiori* conclusions are defeasible, since there may be interferences between the factors: It is possible that certain factors individually favour a certain outcome, but do not favour this outcome in their combination. As a common sense example where this happens, consider the following combination of factors. The fact that the weather is hot may favour the conclusion that one should not go jogging. Similarly, the fact that it is raining also favours the conclusion that one should
not go jogging. However the combination or hot weather and rain can meet one's tastes and indeed be a reason for one to decide to go jogging (for a discussion of this example, see [Prakken and Sartor, 1996]).

### 2.4 From Binary Factors to Dimensions

By playing with factors different patterns of analogical reasoning can be obtained. For example, we can assume that normally by substituting a pro-\(\varphi\) factor \(f\) in a set of factors \(F^{\uparrow \varphi}\) with another pro-\(\varphi\) factor \(f^*\) having a strength which is not inferior to the strength of \(f\), one should obtain a set of factors \(F^{\uparrow \varphi}\) which has a strength which is not inferior to the strength of \(F^{\uparrow \varphi}\).

Assume that a new case \(New_3\) is exactly equal to a previous case \(Prec\), except that \(New_3\) exemplifies \(f^*\) rather than \(f\). If \(Prec\) was decided for \(\varphi\), then \(a\ fortiori\ \ New_3\ should also have decision \(\varphi\).

We cannot here explore the multifarious patterns of analogical reasoning we can obtain by using factors. We shall rather focus on the relationship between binary factors and dimensions (scalable factors).

Categorising a situation as exemplifying or not certain binary factors may be a superficial way of understanding how the features of that situation favour a certain outcome: The binary categorisation may result from transforming a deeper dimensional structure into a binary alternative.

Consider for example binary factor \([\text{long stay}^\uparrow]\). When looking at a case through this category we are only able to say whether the employment contract abroad has a long duration or not; all long duration being valued in the same way. On the contrary, a more refined analysis of the situation would lead us to identify a continuous dimension, the duration of the stay abroad, denoted as \([\text{duration of stay}]\):

- the longer the stay, the more a change decision is favoured,
- the shorter the stay, the more a no-change decision is favoured.

The dimension \([\text{duration of stay}]\) favours a change or a no-change decision with a strength that varies continuously, along the duration of the stay. There is a turning point or threshold-level 0, on the \(y\)-axis, which corresponds to a duration of two years, where the dimension switches direction. Figure 1 shows how, as the stay abroad gets longer, it increasingly favours the change in fiscal domicile.

![Figure 1: The [duration of stay] dimension](image)

Note that Figure 1 represents a linear relationship: It shows a fixed proportion between the quantity of the dimension's property (the duration of the stay) and the strength of the dimension's propensity (its ability to favour change). This is not
generally the case---we would have to draw a curved line, rather than a straight one, to provide a more accurate representation of this connection---but for our purposes a linear relationship is an adequate approximation.

Besides continuous dimensions, there may be discrete dimensions, concerning a property that may only assume discrete values. For example, higher marks in exams may increasingly favour giving a grant to a student, or the number of children in a family may increasingly favour giving the family an allowance.

Moving to a binary representation, we obtain a coarser and simplified view of the matter, where a gradual increase is transformed into a yes/no question: All stays below a certain threshold (one year and a half) are equalised as instances of the [short stay] factor, while all stays above a certain duration (two years and a half) are equalised as instances of the [long stay] factor (see Figure 2). The durations in between one year and a half and two years and a half are equalised as being indifferent to the change issue: They promote neither change nor no-change.

![Figure 2: The [duration of stay] factor](image)

The transformation of dimensions into factors pertains to the strategies of bounded rationality: It is a way of simplifying the analysis of complex situations in order to make them tractable. In particular, looking at a set of cases as instantiating or not instantiating certain factors allows one to perceive at a glance their relevant commonalities and differences. The need that lawyers look at social reality in a tractable way, refraining from impracticable distinctions is a real one, which has inspired many legal theorists, from François Gény see [Gény, 1924] to Oliver Wendell Holmes. However, the conclusion one may reach by only looking at binary factors may be strengthened or questioned by going back to the finer grid of the underlying dimensions.

For better analysing this issue let as add the new dimension [Domestic Assets], which is represented in Figure 4: As the value of the assets (indicated on the x-axis) one has in one's own country increases, the support for a change in the fiscal domicile decreases. The turning point is located at EUR 100,000.
Let us analyse the example cases of Table 4. Consider Precb, which was decided for change, and where the taxpayer had a 4-year work contract and assets of 140,000. It seems that we may extract two messages from this decision:

1. level 4 along [duration of stay] is sufficient to produce change, and
2. this level is sufficient to outweigh the pressure against change provided by having EUR 140,000 of [domestic assets].

Let us now consider the other cases in Table 4:

- Case Newb1 has an easy answer with regard to Precb: Since in Newb1 the duration of the stay abroad is longer (and gives a stronger pull toward change) while the assets remain the same as in Precb, a fortiori Newb1 should be decided for change (in the table we abbreviated a fortiori with a.f.).

- Case Newb2 is covered by a fortiori, with reference to Precb: In Newb2 the same pull for change as in Precb is contrasted by a smaller pull for no-change, due to the smaller level of the domestic assets (120,000 rather than 140,000).

- Case Newb3 can be distinguished form Precb: In Newb3 a higher amount of domestic assets (150,000 rather than 140,000) provides a stronger pull for no-change. On the basis of Precb we cannot tell whether a 4 years duration of the stay abroad still prevails and dictates the outcome. Thus the analogy with Precb can be challenged.

- Case Newb4 can also distinguished from Precb: Newb4 provides a smaller pull for change, due to the inferior duration of the stay abroad (3 years rather than 4), and also a smaller pull for no-change, due to the inferior amount of domestic assets (120,000 rather than 140,000). The precedent does not tell us which of these tendencies is going to prevail.
<table>
<thead>
<tr>
<th>Cases</th>
<th>Duration of stay abroad</th>
<th>Amount of domestic assets</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precₐ</td>
<td>4 years</td>
<td>EUR 140,000</td>
<td>change</td>
</tr>
<tr>
<td>Newₐ₁</td>
<td>5 years</td>
<td>EUR 140,000</td>
<td>change a.f.</td>
</tr>
<tr>
<td>Newₐ₂</td>
<td>4 years</td>
<td>EUR 120,000</td>
<td>change a.f.</td>
</tr>
<tr>
<td>Newₐ₃</td>
<td>4 years</td>
<td>EUR 150,000</td>
<td>distinguish</td>
</tr>
<tr>
<td>Newₐ₄</td>
<td>3 years</td>
<td>EUR 120,000</td>
<td>distinguish</td>
</tr>
</tbody>
</table>

Table 4: Dimensional comparison

Assume now that the dimension [domestic assets] is factorised as shown in Figure 4, that is, into factors [negligible domestic assets↑], spanning from 0 to EUR 75,000, and [substantial domestic assets↓], covering the span above EUR 125,000.

![Figure 4: The [negligible domestic assets] and [substantial domestic assets] factors](image)

Let us now look at the cases in Table 4 according to the grid provided by the corresponding factors: [long stay], [short stay], [negligible domestic assets], [substantial domestic assets]. We get the representation of Figure 5. Note that the chances of distinguishing have been lost: The fact that a dimension (for instance, [duration of stay]) is satisfied to different levels in two different cases (for instance, the fact that the duration is 4 years in one case and 3 years in the other), does not matter, as long as the same factor ([long duration]) applies to both. Consequently, some cases of a fortiori reasoning we provided according to the dimensional representation have now become cases of identity of factors (as for case Newₐ₂), while some possibilities of distinguishing (as for cases Newₐ₃ and Newₐ₄) are now lost.

Only when the difference in the level of a dimension determines the application of a different factor, does this difference become relevant to a factor-based perspective. Case Newₐ₄ is particularly interesting in this regard. According to the factorial representation the inferior duration of the stay, which founded the distinction from Precₐ (and justified not deriving the conclusion change, because of the inferior pull for change) has become irrelevant. On the other hand, the inferior amount of domestic assets, which provides a stronger pull for change is relevant, since is leads to the non-application of the con-change factor [substantial domestic assets] (which was not satisfied in Precₐ). In conclusion, Newₐ₄ satisfies the same pro-change factor [long stay], which was in the Precₐ, but fails to satisfy the con-change factor [negligible domestic assets]; thus it should a fortiori have decision change.
This example shows how moving from dimension to factors pertains to ampliative reasoning: It consists in a cognitive *jump* (see [Peczenik, 1996]), based upon the unstated assumption that differences between dimensional levels are irrelevant, when covered by the same factor. The ability of making such jumps (and reason accordingly, with the advantages of binary reasoning), and of challenging them (on the basis of a dimensional analysis) is a very important aspect of legal rationality. The challenge may include the proposal of a different way of factorising the same dimension (see [Sartor, 2005], chapter 29).

### 2.5 Dimension-Based Inference

Let us try to specify an inference schema similar to the one we have devised for factors, but applying to dimensions and allowing for the refinements that are required for capturing the argument moves we have just exemplified.

As we have observed, each dimension is characterised by the fact that it tends to promote certain outcomes to a certain extent, when it is satisfied up to a certain level. Thus, to describe a situation in terms of certain dimensions we need not only to specify what dimensions apply, but also to what level.

For example, in case of the dimension [duration of stay] we need to indicate how long the stay will be, and in the case of [domestic assets] we need to express the importance of such assets. Various scales may be applied, either discrete or continuous.

Let us introduce some notions that may facilitate our analysis of dimensions.

**Definition 1** Rightward dimensional outcome. We say that outcome O is the rightward outcome with regard to dimensions d, and write \( O_{d}^{\rightarrow} \), if O is increasingly favoured as d’s level increases.

The denomination rightward outcome corresponds to the assumption that the quantity of a dimension is represented along the x-axis, from left to right, as in Figure 1 and Figure 3. For instance, we may say that change is the rightward outcome of dimension [duration of stay] since, as the duration of the stay abroad increases, the outcome change is more strongly favoured.

**Definition 2** Leftward dimensional outcome. We say that outcome O is the leftward outcome with regard to dimensions d, and write \( O_{d}^{\leftarrow} \), if O is increasingly favoured as d’s level decreases.

For instance, we may say that no-change is the leftward outcome of dimension [duration of stay], since, as the duration of the stay abroad decreases, the outcome no-change is more strongly favoured. Many dimensions can also be characterised by a pair of complementary outcomes, such that \( O_{d}^{\rightarrow} \), the rightward outcome of d, is increasingly favoured by increasing degrees of d, and \( O_{d}^{\leftarrow} \), the leftward outcome, is increasingly favoured by decreasing degrees of d. For example, with regard to dimension [duration of stay], change is the rightward outcome and no-change is the leftward outcome. On the contrary, with regard to the dimension [domestic assets], no-change is the rightward outcome and change is the leftward outcome.

Assigning directions to outcomes (with regard to dimensions) allows us to specify when one outcome is more strongly supported according to a dimension.

**Definition 3** Dimensional support (along a dimension). For any couple of dimensional levels \( l_{1} \) and \( l_{2} \) of dimension d, \( l_{1} \) more strongly supports outcome O than \( l_{2} \), if:

- \( l_{1} > l_{2} \), in case that \( O_{d}^{\rightarrow} \), and
- \( l_{1} < l_{2} \), in case that \( O_{d}^{\leftarrow} \).

The notion of a dimensional support allows us to define the comparative strength of sets of dimensions, at least in the uncontroversial case where one set is better with regard to one dimensions, and not worse with regard to any other (and there is no interference between dimensions).
Definition 4 Dimensional strength (of sets of dimensions). Given a set of dimensions $\Delta = d_1, \ldots, d_n$ and two sets $D'$ and $D^2$ of dimensional levels on $\Delta$, $D'$ is dimensionally stronger than $D^2$, with regard to outcome $O$ if

- there is at least one dimension $d_i$, such that $d_i$'s level in $D'$ more strongly supports $O$, then $d_i$'s level in $D^2$ does;
- for every other dimension $d_j$, $d_j$'s level in $D'$ does not support $O$ more strongly than $d_j$'s level in $D^2$ does.

According to this definition, given a set of dimensional levels favouring outcome change, if we substitute a stay abroad of 2 years with a stay of 3 years, all the rest remaining equal, we obtain a stronger pull towards a change decision. The same result would be achieved by changing the taxpayer's domestic assets from EUR 100,000 to EUR 50,000.

2.6 Dimensional A-Fortiori Reasoning

According to the ideas we introduced in the previous section, a dimension-based approach allows for subtler ways of reasoning a fortiori. We can indeed provide the following characterisation of dimensional a fortiori, namely, the outweighing relations between sets of dimensional levels, where $D, D', H$ and $H'$ denote sets of dimensional levels over the same dimensions.

Reasoning schema: Dimensional a fortiori (outweighing)

- $D\uparrow^\phi$ outweighs $H\downarrow^\phi$;
- $D'\uparrow^\phi$ is at least as dimensionally strong as $D\uparrow^\phi$;
- $H'\downarrow^\phi$ is not dimensionally stronger than $H\downarrow^\phi$

IS A DEFEASIBLE REASON FOR

- $D'\uparrow^\phi$ outweighs $H'\downarrow^\phi$

In Table 6 you can see the dimensional evaluation of three of the cases we considered in Table 4, and of which we proposed a factor-based evaluation in Table 5.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Duration of stay</th>
<th>Domestic assets</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prec$_b$</td>
<td>4 years</td>
<td>EUR 140,000</td>
<td>change</td>
</tr>
<tr>
<td>New$_{b1}$</td>
<td>5 years</td>
<td>EUR 140,000</td>
<td>change a.f.</td>
</tr>
<tr>
<td>New$_{b2}$</td>
<td>4 years</td>
<td>EUR 120,000</td>
<td>change a.f.</td>
</tr>
</tbody>
</table>

Table 6: Dimensional a fortiori

Precedent Prec$_b$ had decision change, with regard to a taxpayer who was to be abroad for 4 years and had domestic assets for EUR 140,000. The message of Prec$_b$ is that staying abroad 4 years supports change to such an extent as to outweigh the extent in which having domestic assets for EUR 140,000 supports no-change. Thus case New$_{b1}$, where one stays abroad for 5 years (providing a stronger pull toward change), and has domestic assets as in Prec$_b$ (EUR 140,000) should a fortiori have decision change. The same conclusion should hold in case New$_{b2}$, where the taxpayer stays abroad for 4 years like in Prec$_b$, but holds assets for EUR 120,000 (which provides a lesser pull toward no-change than in Prec$_b$).

3 Conclusion

We hope that our presentation may suffice to show both the importance and pervasiveness of factor-based reasoning in legal argumentation, and its irreducibility to other forms of legal reasoning, like rule-based syllogism, or teleological inference. We believe that indeed the analysis of factor-based reasoning allows us to better understand many important aspects of legal reasoning. Here we have considered some of such aspects, though we certainly could not exhaust the discussion of factor-based reasoning in the law. For the analysis of further aspects of factor-based reasoning, like reasoning with cases and constructing legal theories, see [Sartor, 2005, chapters 28 and 29].
References

[Alexy 1985]

[Ashley 1990]

[Ashley and Rissland 1988]

[Atienza and Ruiz Manero 1998]

[Bench-Capon and Rissland 2001]

[Bench-Capon and Sartor 2003]

[Dworkin 1977]

[Gény 1924]

[Hart 1983]

[Hassemer 1968]

[Kaufmann 1965]

[Larenz 1992]

[McCarty 1982]

[Nozick 1991]

[Peczenik 1996]

[Pound 1954]

[Prakken and Sartor 1996]

[Prakken and Sartor 1998]

[Rissland and Ashley 1987]

[Sartor 2005]

[Sunstein 1996]
Footnotes:

1. This article reports ideas which are discussed and developed in [Sartor 2005]. We refer to the latter work for the theoretical framework in which our analysis of factors is embedded, and for further references to the relevant literature.

2. See also, for a recent discussion of the idea of a dimensions, [Bench-Capon and Rissland, 2001]. On dimensions as scalable factors, see [Bench-Capon and Sartor, 2003].

3. Reasoning with factors, is specifically addressed by [Sunstein, 1996].

4. An assimilation of principles to values (or goals) is proposed by [Alexy, 1985, 75-7], who views principles as commands to optimize, which prescribe to reach a certain outcome as much as possible.

5. Principles are characterised in a rule-like way in [Nozick, 1991]. The assimilation of principles to defeasible rules is assumed by [Atienza and Ruiz Manero, 1998], who view principles as rules having open conditions of applications.

6. On family resemblances, see [Wittgenstein, 1974, secs. 67ff.], who introduced this idea and [Hart, 1983, 174-275], who applied it to the law.

7. On the relation between factors and dimensions, see [Ashley and Rissland, 1988]; [Bench-Capon and Rissland, 2001]; for a discussion of the way of passing from the ones to the others, see [Bench-Capon and Sartor, 2003].