BOUNDED RATIONALITY AND REPEATED NETWORK FORMATION*

SYLVAIN BEAL⋆, NICOLAS QUÉROU⋆

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Abstract: We define a finite-horizon repeated network formation game with consent, and study the differences induced by different levels of individual rationality. We prove that perfectly rational players will remain unconnected at the equilibrium, while nonempty equilibria are possible when, following Neyman (1985), players are assumed to behave as finite automata. We define two types of equilibria, namely the Repeated Nash Network (RNN), in which the same network forms at each period, and the Repeated Nash Equilibrium (RNE), in which different networks may form. We state a sufficient condition under which a given network may be implemented as a RNN. Then, we provide structural properties of RNE. For instance, players may form totally different networks at each period, or the networks within a given RNE may exhibit a total order relationship. Finally we investigate the question of efficiency. We characterize efficient outcomes and prove that the sets of Bentham and Pareto efficient outcomes are identical.

Key words: Repeated network formation game, Two-sided link formation costs, Bounded rationality, Automata.

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⋆CREUSET, University of Saint-Etienne, 42023 Saint-Etienne, France. E-mail adresses : sylvain.beal@univ-st-etienne.fr (corresponding author) and nicolas.querou@univ-st-etienne.fr. Tel : (+33) (0)4 77 42 19 70. Fax : (+33) (0)4 77 42 19 50.

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1 Introduction

Both network structures and rationality of agents play a significant role in determining the outcome of many economic economic relationships. A vast and recent literature examines how network structure affects economic outcomes.[1] The literature on bounded rationality has become more and more important since its introduction by Simon (1955). Our aim is to study the process of network formation within a dynamic framework in two cases related to different levels of rationality for economic agents. In the first one, the agents are perfectly rational. In the second one, some aspects of their rationality are limited.

We consider a group of agents who are initially unconnected who form or remove links with each other. A link can be removed unilaterally but agreement by both agents is needed to form a link. Precisely, a player pays an amount $c > 0$ to seek contact with an opponent and the link forms if the opponent behaves likewise. An agent's payoff is determined as in Gilles and Sarangi's (2004) model with consent. Agents receive the same value from all direct and indirect connections. The cost to a player of creating or maintaining a link is greater than the reward of a single direct connection as in Watts (2002).

Since agreement is required to form links, it is crucial to distinguish an action profile, which lists the wishes or efforts of the players, from the induced network. In fact, several distinct action profiles may lead to an identical network. We focus on a particular subset of action profiles called *cost-efficient action profiles*. In such an action profile, if no link connects two players i and j, then neither i nor j seeks contact with the opponent to create the link. In the static network formation game, only cost-efficient action profiles are likely to define Nash equilibria (NE) since a player incurs a cost for seeking contact with an opponent. For the same reason, only cost-efficient action profiles are likely to be Bentham or Pareto efficient. Precisely, we show that in a NE, perfectly rational players must choose the cost-efficient action profile that induces the empty network. In the finitely repeated network formation game, perfectly rational players also remain unconnected. The unique NE consists in a sequence of cost-efficient action profiles such that the empty network forms in each period. In the current paper, we choose to focus on a finite horizon of play in which a network is built in each period. If players are individuals or firms with a finite lifetime, it seems reasonable to suppose that the relationships they establish stop after a certain amount of time. It

[1] Different examples may be found in consumer's theory (Ellison, Fudenberg, 1995), labor market (Calvó-Armengol, Jackson, 2004), industrial organization (Bolton, Dewatripont, 1994), or in game theory (Ellison, 1993).

is assumed that players create a network in each period. Thus there are many types of possible behaviors for a given player, which depend on the opponents' behavior. Networks may be entirely rebuilt from one period to another, some former links may be maintained or removed, or new links may be created.

In this paper, we also limit players with respect to the complexity of the strategies they can implement. We model this by assuming that they use finite automata with a bounded number of states to play their strategies as in Neyman (1985). Such players are boundedly rational in the sense that they are limited in their ability to count the number of periods played and thus may not anticipate the opponents' behaviors against their own actions. However they have the ability to optimize their payoff against the opponents' strategies given that their choice is reduced to strategies played by finite automata. We also restrict the analysis to cost-efficient action profiles. A player cannot use an established contact with an unconsenting opponent in a period as a message for a possible agreement in a subsequent period. Notice that any network is induced by a unique cost-efficient action profile. Therefore, the restriction to cost-efficient action profiles does not limit the architecture of static networks that may form in an equilibrium outcome. We show that if the size of automata is smaller than the duration of the game, then the set of NE of the repeated game is not reduced to outcomes filled with empty networks, as it is the case with perfectly rational agents. We do not explore whether equilibrium outcomes are affected when the analysis is extended to all action profiles. Nevertheless, cost-efficient equilibria induce a large variety of sequences of networks, which deserve attention.

We distinguish two types of equilibria. In the first one, the same network is formed in all periods. We refer to such an equilibrium as a Repeated Nash Network (RNN). We provide a sufficient condition for the existence of RNNs based on any static network (proposition 3). Moreover, we give a practical test that determines if nonempty RNNs do exist (proposition 4). In the second one, we define a Repeated Nash Equilibrium (RNE) where different networks may form in the different periods of the game. The set of RNE includes the set of RNN as a special case. We show that there exist structural relationships between the different networks that form within a given RNE. We study the intertemporal consistency between networks and identify several properties. Proposition 5 exhibits some sequences of networks that cannot be achieved as an outcome of a nonempty RNE. For instance, a sequence of expanding connected networks cannot constitute an equilibrium outcome, or it is not possible that all players remain isolated during the last two periods. Nevertheless, these restrictions allow for several RNE with nonempty outcomes. The networks within a given RNE may exhibit a total order relationship, the smaller network being formed in the last stage. In particular, there exist RNE in which sequences of contracting networks form, or in which sequences of networks can expand for all but the last period in which connections brutally run low. We also show that there are RNE in which players forget links (proposition 6). Precisely, two directly connected players in one period are not directly connected in another period. In spite of the restrictions on the intertemporal consistency between networks in equilibrium outcomes, we prove that any network can emerge in the outcome of a RNE (proposition 7). We also investigate the question of efficiency for both Bentham and Pareto criteria. In the finitely repeated game, the structure of efficient strategy profiles is closely related to the structure of static efficient networks (propositions 8 and 9). In addition, we prove that the sets of Bentham-efficient networks (strategy profiles) and Pareto-efficient networks (strategy profiles) are identical. The set of efficient strategy profiles of the repeated game is most often slightly reduced when players are assumed to be boundedly rational (proposition 10).

Some other papers are concerned with the structural properties of NE in repeated games with finite automata.[2] Papers relating to repeated games are mainly concerned with the set of average payoffs that can be achieved in RNE. In the current work, we are more interested in the structure of RNE than in the induced average payoffs. The differences with other papers studying a dynamic network formation[3] are mostly related to the aims of the studies. These papers are concerned with the formation of a static network as a result of several steps of a dynamic process. Limiting networks are studied in Watts (2001), and learning or stochastic stability are used to identify limiting equilibria in Bala, Goyal (2000). By contrast, we consider a finitely repeated game that consists in the formation of a static network in each step of the process. We are interested in understanding the influence of different levels of rationality on equilibrium structures in this finite-horizon repeated setting.

The rest of the paper is organized as follows. Section 2 presents preliminaries and notations. We also determine the set of Nash networks in the static game and in the finitely repeated game for the case of perfectly rational players. Then the machine game is introduced and studied in section 3. We start with results on RNN and continue with results on structural properties of more elaborated RNE. Results dealing with the efficiency of networks and strategy profiles are in section 4. Once again, we distinguish results according to players' rationality. Section 5 concludes. Proofs not given in the body

[2]See Rubinstein (1986), Abreu, Rubinstein (1988) and Piccione, Rubinstein (1993) among others.

[3]We refer the reader to Bala, Goyal (2000), Currarini, Morelli (2000), Dutta, Ghosal, Ray (2005), Goyal, Vega-Redondo (2005), Jackson, Watts (2002) and Watts (2001, 2002).

of the paper appear in an appendix.

2 Preliminaries and notations

2.1 Static one-period game

Let $G = (I, A, \pi)$ be a finite n-player game in normal form. The set $I = \{1, \ldots, n\}$ is the player set. For any $i \in I$, A_i is player i's set of actions and $A = \prod_{i \in I} A_i$ is the set of action profiles. Let a_{-i} be the actions chosen by all players except i and $A_{-i} = \prod_{j \neq i} A_j$. Player i's payoff function is $\pi_i : A \longrightarrow I\!R$. An action profile a is a NE of G if for any $i \in I$ and any $a'_i \in A_i$,

$$\pi_i(a) \geq \pi(a'_i, a_{-i}).$$

2.2 Network

The n players are connected in some network relationships. We limit our discussion to non-directed networks on the player set I. As in Jackson, Wolinsky (1996), two players are either related to each other or not, but it cannot be that one is related to the second without the second being related to the first. We write ij to describe the *link* between two players i and j.

Let $g_I = \{ij : i, j \in I, i \neq j\}$ be the set of all potential links. Any set of links $g \subseteq g_I$ defines a *network*. We apply the convention that $g = g_I$ is the *complete* network and that $g = g_0 = \{\emptyset\}$ is the *empty* network. Any (spanning) subset $g' \subset g$ is called a *subnetwork* of g.

A *path* between players i and j in a network g is a sequence of distinct players $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ where $i_1 = i$ and $i_K = j$. Two such players are said to be connected. Player i is in a *cycle* of network g if there is a path with $K \geq 3$ players such that $i_1 = i_K = i$.

Let $n_i(g) = \{j \in I | ij \in g\}$ be the set of *neighbors* (or direct connections) of player i. Let $N_i(g)$ be the set of players to whom player i is connected in network g. Obviously, $n_i(g) \subseteq N_i(g)$. A network g is connected if there is a path between any two players. Alike, network g is said to be k-connected if there does not exist a set of $k-1$ links whose removal disconnects the network. If g is not connected, its connected subnetworks are called *components*. A connected acyclic network (or 1-connected network) is called a *tree* and a non connected network whose distinct components are trees is called a *forest*.

Let $l_{i,j}(g)$ be the *distance* between two players i and j in network g. If i and j are connected, $l_{i,j}(g)$ is the number of links in the shortest path between i and j. By convention, if i and j are not connected, $l_{i,j}(g) = \infty$. Let $L_i(g) = \max_{j \neq i} l_{i,j}(g)$ be player i's *eccentricity* in network g. The *diameter* of network g is $L(g) = \max_{i \in I} L_i(g)$. The last two definitions apply to any component $g' \subset g$.

For any two distinct players $i, j \in I$, $g + ij = g \cup \{ij\}$ is the network obtained adding link ij to network g. Likewise, let $g - ij = g \backslash \{ij\}$ be the network obtained removing link ij from network g. The intersection $g \cap g'$ defines the set of links that networks g and g' have in common.

2.3 Link formation cost and inefficient links

In this section, we present a non-cooperative model of costly network formation with consent. We assign a network $g(a) \subseteq g_I$ to every action profile $a \in A$. Each player $i \in I$ has an action set $A_i = \{(a_{ij})_{j \neq i} : a_{ij} \in \{0, 1\}\}$. Player i seeks contact with player j if $a_{ij} = 1$. Link ij forms if both players seek contact. The network induced by a is given by

$$g(a) = \{ij \in g_I : a_{ij} = a_{ji} = 1\}.$$

If player i seeks contact with j, then he supports a cost $c > 0$. As in a wide range of models of costly network formation, player i's payoff consists in his value of the network minus a cost c for any attempt he made to create links. We assume that the value of network $g(a)$ for player i only depends on the number $\#N_i(g(a))$ of players to whom i is connected where $\#$ gives the dimension of the set. Thus, the distance between two connected players does not matter. This is true of many networks such as Internet, economic partnership or subcontracting. This results in the following payoff function

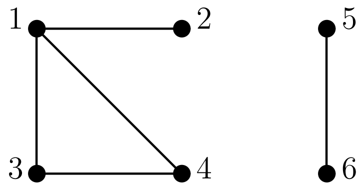
$$\pi_i(a) = v\#N_i(g(a)) - c \sum_{j \neq i} a_{ij}, \tag{1}$$

which can be seen as a particular case of the class of payoff functions investigated by Gilles, Sarangi (2004). Following Watts (2002), we assume $c > v > 0$, that is, creating a link is more costly than the reward of a single direct connection. In other words, player i needs some indirect connections to obtain a positive payoff. We also fix an upper bound $2v > c$ to avoid trivial cases due to an immeasurable cost of creating a link. The next example will help discussing these assumptions.

Example 1
Consider $I = \{1, 2, 3, 4, 5, 6\}$ and the network that forms as a result of the following players' choices:

$a_{ij} \backslash a_{ji}$	1	2	3	4	5	6
1	0	1	1	1	0	0
2	1	0	0	0	0	0
3	1	0	0	1	0	0
4	1	0	1	0	0	0
5	0	0	0	0	0	1
6	0	0	0	1	1	0

a



$g(a)$

Figure 1.

Network $g(a)$ consists in two components. The value of $g(a)$ is identical for players 1, 2, 3 and 4 in the left component and their payoffs only differ in the cost supported. Player 2's payoff is thus larger than 1's payoff since 2 intends to form a link with 1 while 1 intends to form links with 2,3 and 4. The subset $\{1, 3, 4\} \subset I$ defines a cycle in $g(a)$. Since the distance between two connected players has no influence on payoffs, it is not the interest of player 1 to seek contact with 3 since they are already connected by player 4. Intuitively, link 13 is superfluous. Now consider the right component of $g(a)$ involving players 5 and 6. This component is a tree, but $g(a)$ is not a forest since the right component includes a cycle. The assumption $c > v$ implies that both 5 and 6 would gain to remain isolated as it is the case in all components of diameter 1. Remark also that 6 seeks contact with 4. The link 64 fails to form since $a_{46} = 0$. As player 6's attempt is not drawn on $g(a)$ even if it affects his payoff, we will distinguish the action profile a and its induced network $g(a)$. $\square$

Two identical networks $g(a) = g(b)$ may correspond to distinct action profiles a and b. Precisely, $g(a) = g(b)$ for two action profiles $a \neq b$ across the player set I if both networks have the same set of links and if at least one player seeks the creation of a link but does not receive the consent of the opponent. We will focus on a particular form of action profiles, namely the *cost-efficient* action profiles.

Definition 1 *An action profile a is cost-efficient if there is no $(i, j) \in I^2$ such that $a_{ij} \neq a_{ji}$.*

Let A^{ce} be the set of cost-efficient action profiles. In any cost-efficient action profile, if a link between players i and j fails to form, neither i nor j seeks contact to create it. The only cost-efficient action profile that enables the empty network to form is denoted $a_0 = (a_{1,0}, \dots, a_{n,0}) \in A^{ce}$ where $a_{i,0} = (0, \dots, 0)$. Clearly, the action profile a in example 1 is not cost-efficient.

If the payoff function is given by (1) and $2v > c > v > 0$, player i may obtain a larger payoff if he removes some links. Abusing notations, if $ij \in g(a)$, let $a_i(j^-)$ be the action that differs from a_i only by $a_{ij} = 0$.

Definition 2 *A link $ij \in g(a)$ is superfluous for player i in network $g(a)$ if $\pi_i(a_i(j^-), a_{-i}) > \pi_i(a)$. This inequality is satisfied in two cases:*

1. *$N_i(g(a) - ij) = N_i(g(a))$, i.e. link ij belongs to a cycle;*
2. *$n_j(g(a)) = \{i\}$, i.e. player i is j's single neighbor.*

In case 1, the removal of ij does not alter player i's connection set. This is the case of link 13 in example 1. In case 2, link ij increases the value of the network but costs to player i more than it yields since it provides him with a single connection and $c > v$. In example 1, this is the case of link 56 for both players 5 and 6. Let $d_i^1(a)$ (resp. $d_i^2(a)$) denote the total gain for player i that results from removing all superfluous links in case 1 (resp. case 2). We have

$$d_i^1(a) = c\left(\#\{j \in I : N_i(g(a) - ij) = N_i(g(a))\} - \mathcal{C}_i\right), and$$

$$d_i^2(a) = (c - v)\#\left\{ij \in g(a) : n_j(g(a)) = \{i\}\right\},$$

where $\mathcal{C}_i$ is the number of cycles with i as unique common player. Now let $d_i(a) = d_i^1(a) + d_i^2(a)$ be the maximal gain for player i if he switches from action a_i against a_{-i}.

For any $i \in I$, we define $b_i \in A_i$ as player i's best response against a_{-i}, that is

$$b_i \in \arg\max_{a_i' \in A_i} \pi_i(a_i', a_{-i}).$$

When $a \in A^{ce}$, b_i is the action that satisfies

$$\pi_i(b_i, a_{-i}) - \pi_i(a) = d_i(a),$$

since a player cannot create links by his own will. All these definitions and notations will be used in the rest of the paper to determine whether action profiles are NE or efficient. Now we begin by characterizing Nash networks.

Proposition 1 *The only Nash network of G is the empty network $g_0 = g(a_0)$.*

A direct consequence of proposition 1 is that any player $i \in I$ can secure a null payoff against any opponents' behaviors by choosing $a_{i,0}$. The *minmax* payoff of each player is then 0. Now we introduce the finitely repeated game.

2.4 Finitely repeated game and network formation

In the static game G, players will not create any link even if they would be better off in some nonempty networks. In this section we assume that the players are involved in a T-period repeated game G^T that consists in

$T < \infty$ repetitions of game G in period $t = 1, 2, \ldots, T$. Throughout the paper, we assume $T > 3$. It is natural to consider a dynamic process of network formation. Such a framework fits with many economic situations in which relationships between agents may evolve with time.

We take the view that at the beginning of period t, all players observe a^{t-1} and not just $g(a^{t-1})$. If network $g(a)$ in example 1 forms in period $t-1$, then all players know at the beginning of period t that player 6 has contacted player 4 even if link 64 fails to form.

Thus, a history of play $h^t = (a^1, \ldots, a^{t-1})$ at period t records the action profiles chosen by each player in periods $1, \ldots, t-1$. Let H^t denote the set of histories at period t and $H = \{\emptyset\} \cup (\bigcup_{t=2}^{T} H^t)$ denote the set of all possible histories of play. A repeated game strategy s_i is a sequence $s_i = \{s_i^t\}_{t=1}^{T}$ where $s_i^t : H^t \longrightarrow A_i$ models player i's action played at period t as a function of the $t-1$ previous action profiles. For any $i \in I$, S_i is the set of strategies for player i. Let $s = (s_1, \ldots, s_n)$ denote a strategy profile and $h^{T+1}(s) = (a^1, \ldots, a^T)$ be the repeated game outcome induced by $s \in S$, where $S = \prod_{i \in I} S_i$ is the set of all strategy profiles.

Player i's payoff function $\tilde{\pi}_i : S \longrightarrow I\!R$ from playing G^T is evaluated according to the average payoff

$$\tilde{\pi}_i(s) = \frac{1}{T} \sum_{t=1}^{T} \pi_i(a^t), \tag{2}$$

and $\tilde{\pi} = (\tilde{\pi}_1, \ldots, \tilde{\pi}_n)$. A strategy profile $s \in S$ is a NE of the repeated game G^T if, for each $i \in I$ and $s_i' \neq s_i$,

$$\tilde{\pi}_i(s) \geq \tilde{\pi}_i(s_i', s_{-i}).$$

Since a network is built in each period of the repeated game, it is useful to introduce the notion of *repeated network*.

Definition 3 *A strategy profile s^* induces a repeated network based on network $g(a^*)$ if s^* induces a repeated game outcome $(a^1, \ldots, a^T)$ that satisfies $g(a^t) = g(a^*)$, $\forall t = 1, \ldots, T$.*

A repeated network is simply a network that forms in all periods as a result of players' actions. We want to highlight the robustness of a network being formed period after period. This notion is intuitively related to the robustness of a given set of relationships. We may think about situations in which trust is established among agents on a long-term basis. A Repeated Nash Equilibrium (RNE) is a NE of the repeated game. We need to define a Repeated Nash Network (RNN).

Definition 4 *A strategy profile s^* is a RNN based on network $g(a^*)$ if the induced outcome is a repeated network based on $g(a^*)$ and if it is a RNE of G^T.*

Thus the set of RNE includes as a special case the set of RNNs. When the horizon is finite, the only RNE induces the formation of the only Nash network in all periods, *i.e.* players always remain isolated.[4]

Proposition 2 *Assume $T < \infty$. The only RNE is the one in which the empty network g_0 forms in all periods.*

In the present section the agents are assumed to be perfectly rational. This results in an extreme conclusion: players have an incentive to remain unconnected. Now, we are going to relax the assumption of perfect rationality in order to understand the resulting differences on the types of relationships that are likely to appear.

3 Machine game in a finite horizon setting

In this section, we assume that players use finite automata with a limited number of states to play their strategies. We also focus on the subset of cost-efficient action profiles. This restriction may be justified by the fact that only cost-efficient action profiles are likely to define Nash networks in the static game. Moreover, for any network structure, the corresponding cost-efficient action profile is the most efficient action profile that induces the network (see section 4). In other words, only cost-efficient action profiles are likely to define efficient networks. We begin with the study of RNNs. Then, we examine RNE which are not RNNs. We will prove that the structure of both types of equilibria becomes non degenerate.

3.1 Machine game

Following Neyman (1985), we focus on the repeated network formation game G^T in which player $i \in I$ chooses a *finite automaton* M_i to play his strategy. A finite automaton M_i for player i is a four-tuple $(Q_i, q_i^1, \lambda_i, \mu_i)$ where

1. Q_i is the finite set of states in M_i, with $\#Q_i = m_i$;
2. q_i^1 is the initial state;
3. $\lambda_i : Q_i \longrightarrow A_i$ is the output function, which plays action $\lambda_i(q_i) \in A_i$ whenever M_i is in state q_i;
4. $\mu_i : Q_i \times A_{-i} \longrightarrow Q_i$ is the transition function. In a given period, if M_i is in state $q_i \in Q_i$ and players $-i$ choose $a_{-i} \in A_{-i}$, then the next state of the machine is $\mu_i(q_i, a_{-i}) \in Q_i$.

[4] When the game is infinitely repeated, we can prove a folk theorem like result for RNE and RNN. Such a result includes a very large panel of structures even if, for instance, there cannot be a RNN based on a star network.

We assume that player i's strategy space is limited to the set $\mathcal{M}_i$ of all automata of size (the number of states in the machine) $1 < m_i < T$. Before proceeding to the results, we must apply these notations to the repeated network formation game. A strategy profile of the machine game is an n-tuple $(M_1, \ldots, M_n)$ of automata. We also use the notation (M_i, M_{-i}) instead of $(M_1, \ldots, M_n)$. Abusing notations, we keep up writing a^t for the action profile chosen in period t instead of using the notation $(\lambda_1(q_1^t), \ldots, \lambda_n(q_n^t))$.

3.2 Existence of nonempty RNN

The main goal of this section is to provide a sufficient condition for a nonempty network to be sustained as a RNN. Before stating the result, notice that there cannot be a RNN based on a star network. In fact, the central player obtains a negative payoff as he supports a cost for creating a link with each opponent.

Proposition 3 *Consider any network $g(a^*)$ such that $a^* \in A^{ce}$ and, for any $i \in I$,*

$$0 \leq d_i(a^*) \leq \min \left\{ \frac{v}{2} \# N_i(g(b_i, a^*_{-i})), 2\pi_i(a^*) \right\}. \tag{3}$$

Then there exists a RNN $(M_1, \ldots, M_n)$ such that $g(a^*)$ forms in each period.

Proof. Suppose that condition (3) is satisfied. Choose any $a^* \in A^{ce}$. We show that there exists a RNN $(M_1^*, \ldots, M_n^*)$ that induces $g(a^*)$ in all periods by constructing the required automata. For any player $i \in I$, let us consider the trigger strategy s_i defined for $t = 1$ by $s_i^1(\emptyset) = a_i^*$, and for $t > 1$, by

$$s_i^t(h^t) = \begin{cases} a_i^* & \text{if } a_{-i}^\tau = a_{-i}^*, \ \forall \tau = 1, \ldots, t-1, \\ a_{i,0} & \text{otherwise.} \end{cases}$$

This strategy is implemented by the two-state automaton M_i^1 represented below:

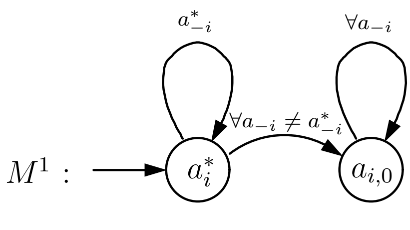


Figure 2.

If the opponents play M_{-i}^1, then player i's payoff from playing M_i^1 is

$$\tilde{\pi}_i(M_i^1, M_{-i}^1) = \pi_i(a^*) \geq 0, \tag{4}$$

since the outcome of the machine game is assumed to be a repeated network.

Notice that any deviation from M_i^1 by player i releases a definitive punishment by players $j \neq i$. Recall that each player i is limited to automata of size $1 < m_i < T$. This prevents player i from using the standard best response against M^1_{-i} that consists in playing a_i^* to create $g(a^*)$ until the last stage and then playing b_i in round T. In fact such a strategy requires at least a T-state automaton. As a consequence, a deviation by player i must occur in a period $t < T$ and implies $T - t \geq 1$ periods of punishment. This also implies that if player i has an incentive to deviate, then this deviation must occur as late as possible in the game. However, as we will see below, this does not exactly amount to say that if player i has an incentive to deviate, he will do so in stage $T - 1$.

Any deviation in stage $t < T$ is followed by some punishment stages. Thus, player i aims at minimizing the cost of seeking contacts in the actions he plays in periods $t+1, \ldots, T$ against $a_{-i,0}$ since the value of the network in each of these periods is null. Even if the amount paid for seeking contacts in the deviating action is less than a_i^*, it may be very costly. Therefore, it may not be the interest of player i to play this action at stage $t+1$ and thereafter. This is why all possible deviations for player i's may be grouped into the two following cases:

1. He plays b_i in periods $T-1$ and T,[5]
2. He plays b_i in period $k \leq T-2$ and uses a $(k+1)$th state that plays $a_{i,0}$ for the remaining stages. Clearly, player i's highest incentive to deviate is in period $k = T-2$.

We now consider these two possibilities.

Case 1.
The deviation is implemented by the following $(T-1)$-state automaton:

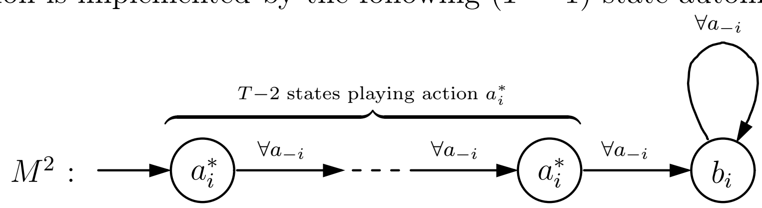


Figure 3.

Using M_i^2, player i mimics a full cooperation to form network $g(a^*)$ up to period $T-2$, and then plays b_i to obtain the best payoff against $a_{-i}^* = a_{-i}^{T-1}$.[6]

[5] Recall that using $T-2$ states for playing a_i^* and one more state for playing b_i prevents player i from playing $a_{i,0}$ in stage T as a response to $a_{-i,0}$. Moreover, player i has not interest in deviating at stage $T-1$ to any action a_i which is less costly than b_i. This is due to the fact that using a_i removes at least two more players from i's set of connections than b_i. Therefore, $\pi_i(a_i, a_{-i}^*) + \pi_i(a_i, a_{-i,0}) < \pi_i(b_i, a_{-i}^*) + \pi_i(b_i, a_{-i,0})$ due to $2v > c$.

[6] Notice that action profiles considered in the deviation tests are not cost-efficient. In fact removing links from a_i^* by playing b_i induces an action profile $(b_i, a_{-i}^*) \notin A^{ce}$ that is not cost-efficient.

He also plays b_i in period T against the punishment $a_{-i,0}$ since

$$b_i = \arg \max_{a_i \in \{b_i, a_i^*\}} \pi_i(a_i, a_{-i,0}) = \arg \min_{a_i \in \{b_i, a_i^*\}} -c \sum_{j \neq i} a_{ij}.$$

In words, player i's machine has not enough states to use another action than b_i or a_i^*. Moreover, the cost of seeking contacts in b_i cannot be more expensive than in a_i^* since the assumption $a^* \in A^{ce}$ prevents the deviating player from creating links unilaterally. Player i obtains the payoff

$$\tilde{\pi}_i(M_i^2, M_{-i}^1) = \frac{(T-2)\pi_i(a^*) + \pi_i(b_i, a_{-i}^*) + \pi_i(b_i, a_{-i,0})}{T}, \tag{5}$$

which has to be compared to (4). It is not the interest of player i to switch from M_i^1 to M_i^2 if and only if

$$\begin{aligned}
& \pi_i(a^*) && \geq \frac{(T-2)\pi_i(a^*) + \pi_i(b_i, a_{-i}^*) + \pi_i(b_i, a_{-i,0})}{T} \\
\iff \quad & \pi_i(a^*) && \geq \frac{v}{2} \# N_i(g(b_i, a_{-i}^*)) - c \sum_{j \neq i} a_{ij}^{*d} \\
\iff \quad & \pi_i(a^*) && \geq \pi_i(b_i, a_{-i}^*) - \frac{v}{2} \# N_i(g(b_i, a_{-i}^*)) \\
\iff \quad & d_i(a^*) && \leq \frac{v}{2} \# N_i(g(b_i, a_{-i}^*)). \qquad (6)
\end{aligned}$$

Case 2.
The deviation is implemented by the following $(T-1)$-state automaton:

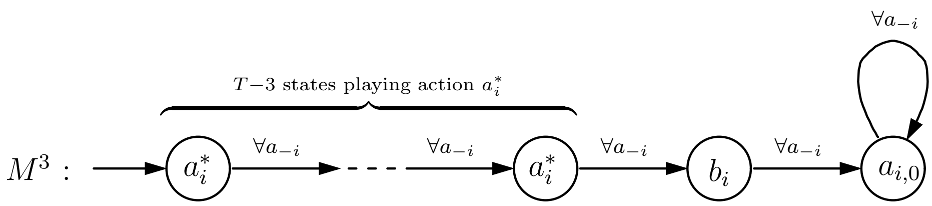


Figure 4.

Using M_i^3, player i simulates a full cooperation to create network $g(a^*)$ up to period $T-3$, and then plays in stage $T-2$ the action b_i against $a_{-i}^* = a_{-i}^{T-1}$. He uses a new state playing $a_{i,0}$ for the last two stages. Thus, player i obtains the average payoff

$$\tilde{\pi}_i(M_i^3, M_{-i}^1) = \frac{(T-3)\pi_i(a^*) + \pi_i(b_i, a_{-i}^*) + 2\pi_i(a_{i,0}, a_{-i,0})}{T},$$

which also needs to be compared to (4). It is not the interest of player i to switch from M_i^1 to M_i^3 if and only if

$$\begin{aligned} & \pi_i(a^*) && \geq \frac{(T-3)\pi_i(a^*) + \pi_i(b_i, a^*_{-i}) + 2\pi_i(a_{i,0}, a_{-i,0})}{T} \\ \iff & d_i(a^*) && \leq 2\pi_i(a^*). \end{aligned} \tag{7}$$

Combining the two cases, player i will not deviate from strategy M_i^1 if and only if

$$d_i(a^*) \leq \min\left\{\frac{v}{2}\#N_i(g(b_i, a^*_{-i})), 2\pi_i(a^*)\right\},$$

the condition stated in the proposition. By definition, $d_i(a^*) \geq 0$ so that inequality (3) guarantees that players obtain at least the minmax payoff. Thus, the strategy profile $(M_1^*, \ldots, M_n^*)$ is a RNN based on $g(a^*)$. ■

Remark that the condition in proposition 3 does not depend on whether T is large or not since the average payoff function does not include a discount parameter. This condition implies that RNNs must be based on networks that are not too much over-connected if players are very rancorous. Indeed, the gain that each player obtains may not be too small. In static network theory, the fact that networks be too much over-connected is important too, mostly for efficiency considerations (Calvó-Armengol, 2003). The condition in proposition 3 also implies that a player must not support a too large share of the cost needed to connect his component. For instance $g(a^*)$ cannot contain a star subnetwork since a player would be directly connected to each opponent and obtains in average less that the minmax payoff.

Condition (3) is necessary and sufficient to prevent any deviation from (M_i^1, M_{-i}^1). However, it is not a necessary condition to achieve a RNN based on $g(a^*)$. To see this, notice that M^1 induces the hardest possible punishment, either for the duration or for the loss of static game payoffs per period. One can think of an automaton that induces a less significant loss of payoff for a single period of punishment. If all players use such a machine and none of them has an incentive to deviate, then the necessary and sufficient condition to achieve a RNN based on network $g(a^*)$ must be less restrictive than (3). We limit our result to the sufficient condition because it is difficult to determine the action for which the threat of punishment is minimal.

Let $g(l)$, $l \in A^{ce}$, be an n-player line network: for any $i \in I\backslash\{h, j\}$, $n_i = 2$ and $n_h = n_j = 1$ and all players have $n-1$ connections. The line begins and ends with players h and j. The next proposition gives two practical tests to determine whether the set of RNN contains nonempty repeated networks. In the first test, it is enough to check if a repeated line network is not a RNN to make sure that any other nonempty repeated network is not a RNN. This test proves useful if one wants to check that there cannot be a RNN based on a large and complex static network. In the second test, it is sufficient to look at the number of players to guarantee that some nonempty RNNs do exist.

Proposition 4 *(i) Suppose that the repeated network based on $g(l)$ cannot be a RNN. Then the only RNN of G^T induces the empty network g_0 in all periods.*

(ii) Suppose $n \geq 5$. Then there exists a nonempty RNN of G^T.

Proof. *(i)* Suppose that there is no automata profile such that the repeated network based on $g(l)$ is a RNN. Firstly, observe that if the value to a player of any network $g(a)$ is more than v, then there is at least one player whose cost is $2c$ or more. Given that, $g(l)$ is the network in which the minimal payoff across the player set is maximal across the set of all networks. Secondly, the players in the line network are connected with a minimal number of links and the cost for creating links is distributed such that a player pays at most $2c$, which cannot be less in a connected network. In other words, the network $g(l)$ is the unique architecture that satisfies the maxmin criterion. That is,

$$\min_{i \in I} \pi_i(l) = \max_{a \in A^{ce}} \min_{i \in I} \pi_i(a). \tag{8}$$

It follows from (8) that if a player has an incentive to deviate from the repeated network based on $g(l)$, then there is at least one player who can do so in a repeated network based on any network $a \in A^{ce}$. The assumption that the repeated network based on $g(l)$ is not a RNN implies that the only RNN of G^T induces a Nash network in all periods. Therefore, the empty network must form in all periods.

(ii) We next prove that $g(l)$ is sustained as a RNN (whatever $2v > c > v$) if $n \geq 5$. We are going to use the sufficient condition of proposition 3. In the cost-efficient action profile that induces network $g(l)$, all players except the first and the last of the line seek the creation of two links (the cost to each of these players in the network is $2c$), while they benefit from connections with all opponents (in fact, the value of $g(l)$ is $v(n-1)$ for all players). For players h and j, the value of $g(l)$ remains $v(n-1)$ as $g(l)$ is connected but they only create a single link. As a consequence,

$$\min_{i \in I} \pi_i(l) = v(n-1) - 2c,$$

In $g(l)$, only the neighbors of h and j have a superfluous link. Let i be h's neighbor. It follows that $b_i = l_i(h^-)$ which implies that $g(b_i, l_{-i})$ consists in an $(n-1)$-player line of extermities i and j and an isolated player h. Player i has $d_i(l) = c - v$. Therefore, $d_i(l) \geq d_j(l)$ for any player $j \neq i$. By condition 3, there is a RNN based on $g(l)$ if

$$c - v \leq \min \left\{ \frac{v}{2}(n-2), 2(v(n-1) - 2c) \right\}.$$

The ineguality $c - v < v(n-2)/2$ is satisfied when $n \geq 4$ and the ineguality $c - v < 2(v(n-1) - 4c)$ is satisfied when $n \geq 5$. Thus, the condition $n \geq 5$ is enough to guarantee the existence of nonempty RNN. ■

So far, two sufficient conditions for the existence of nonempty RNNs are provided. The second one has been stated in the most general form to keep the exposition as simple as possible. We may notice that any line network with at least 5 players can be sustained as a RNN. The results of this section enable to make precise the cases in which nonempty RNNs do exist. But not much is said about the structural properties of RNE that are not RNNs. This is the aim of the next section.

3.3 Structural properties of RNE

In this section, we are mainly concerned with the structural properties of RNE of G^T. We identify a property satisfied by any RNE. This property has a crucial impact on the intertemporal consistency between networks that form in the outcome of RNE. In propositions 5 and 6, we use graph theory to characterize these restrictions and to represent the sequences of networks that can be achieved at equilibrium. We also offer economic interpretations.

We begin with an intuitive property satisfied in any RNE of G^T. There is a key argument in the analysis of the structure of RNE. To see this, suppose that player i uses an action a_i^k in a period $k < T$ that is more beneficial than a_i^T against actions a_{-i}^T used by the opponents in stage T. It is the interest of player i to play this action in the last stage since he cannot be punished in a forthcoming period and a $(T-1)$-state automaton can do it. Therefore, players must not have such an opportunity to deviate in any RNE.

Lemma 1 *Consider any RNE (M_i^*, M_{-i}^*) of the machine game G^T with outcome $(a^{*1}, \ldots, a^{*T})$. Then there is no network $g(a^{*k})$ that forms in period $k < T$ such that for any player $i \in I$, $\pi_i(a_i^{*k}, a_{-i}^{*T}) > \pi_i(a^{*T})$.*

Proof. The proof is by contradiction. Consider a RNE (M_i^*, M_{-i}^*) of G^T, a player i and a period $k < T$ in which network $g(a^{*k})$ forms such that $\pi_i(a_i^{*k}, a_{-i}^{*T}) > \pi_i(a^{*T})$. We prove that player i has an incentive to deviate from M_i^* towards the following $(T-1)$-state automaton M_i:

1. $Q_i = \left\{q_i^{s_1}, \ldots, q_i^{s_t}, \ldots, q_i^{s_{T-1}}\right\}$, $m_i = T-1$;

2. $q_i^1 = q_i^{s_1}$;

3. $\lambda_i(q_i^{s_t}) = a_i^{*t} \in h^{T+1}(M_i^*, M_{-i}^*) = ((a_i^{*1}, a_{-i}^{*1}), \ldots, (a_i^{*T}, a_{-i}^{*T})), \forall t \leq T-1$;

4. $\mu_i(q_i^t, \lambda_{-i}(q_{-i}^t)) = \begin{cases} q_i^{s_{t+1}} & \text{if } t \leq T-2 \\ q_i^{s_k} : \lambda_i(q_i^{s_k}) = a_i^{*k} & \text{if } t = T-1 \end{cases}$

The output function indicates that for each $t < T$ the action used by M_i^* against M_{-i}^* in period t is played when in state $q_i^{s_t}$. The transition function of M_i mimics the sequence of actions played by M_i^* against M_{-i}^* for all but the last period. In any period $t < T$, the deviation towards M_i keeps player i's payoff unchanged. In stage T, M_i transits to the state $q_i^{s_k}$ that implements action a_i^{*k} used by M_i^* against M_{-i}^* in stage k. Using M_i, player i obtains the following average payoff:

$$\tilde{\pi}_i(M_i, M_{-i}^*) = \tilde{\pi}_i(M_i^*, M_{-i}^*) + \frac{\pi_i(a_i^{*k}, a_{-i}^{*T}) - \pi_i(a^{*T})}{T},$$

which is larger than $\tilde{\pi}_i(M_i^*, M_{-i}^*)$ since $\pi_i(a_i^{*k}, a_{-i}^{*T}) > \pi_i(a^{*T})$ by assumption. This contradicts the initial assumption that (M_i^*, M_{-i}^*) is a RNE. ■

The idea that a player may use a former state to deviate in stage T without being punished is central in the question of the architecture of RNE. Lemma 1 states restrictions on the intertemporal consistency between static networks that form within a given RNE of G^T. Unfortunately, these restrictions are described in terms of payoff. We are more interested in structural properties of the networks induced by such RNE. We specify some of these properties in points *(i)* and *(ii)* of the next proposition. The third point is related to both proposition 2 and lemma 1.

Proposition 5 *The outcome $(a^1, \ldots, a^T)$ induced by any RNE $(M_1, \ldots, M_n)$ of game G^T must have the three following features:*

(i) there is no connected network $g(a^t) \subset g(a^T)$, $\forall t < T$,

(ii) in network $g(a^T)$, there is no player i with eccentricity $L_i(g(a^T)) = 1$,

(iii) if there is a period $t < T-1$ such that $g(a^t) \neq g_0$, then it cannot be the case that $g(a^{T-1}) = g(a^T) = g_0$.

Proof. *(i)* The proof is by contradiction. Consider any RNE $(M_1, \ldots, M_n)$ of G^T in which, for some $t \in \{1, \ldots, T-1\}$, a connected network $g(a^t) \subset g(a^T)$ forms (see definition page 4). The assumptions $g(a^t)$ connected and $g(a^t) \subset g(a^T)$ imply of course that $g(a^T)$ is also connected. By definition 2, there is a player i who has some superfluous links in $g(a^T)$ that he does not have in $g(a^t)$. This player is able to play a_i^t in stage T against a_{-i}^T by a mechanism similar to that in proof of lemma 1. In network $g(a_i^t, a_{-i}^T)$ player i maintains a connection with all opponents since $g(a^t)$ is a connected sub-network of $g(a^T)$ (in fact, $\#N_i(g(a_i^t, a_{-i}^T)) = \#N_i(g(a^T)) = n-1$). Moreover the cost to player i of forming links in $g(a_i^t, a_{-i}^T)$ is smaller since he seeks less contacts in a_i^t than in a^T. Thus, if player i chooses the machine that simulates what plays M_i against M_{-i} for all but the last period and then transits to the state used in period t, he obtains a larger payoff in stage T. Player i obtains a larger average payoff, which implies that (M_i, M_{-i}) is not a RNE. This contradicts the initial assumption.

(ii) By contradiction, consider a RNE $(M_1, \ldots, M_n)$ in which $L_i(g(a^T)) = 1$ for a player $i \in I$. Such an eccentricity means that player i is directly connected with each opponent. This implies that player i obtains the worst possible stage payoff $\pi_i(g(a^T)) = (n-1)(v-c)$. Since $(M_1, \ldots, M_n)$ is a RNE, we know that $\tilde{\pi}_i(M_1, \ldots, M_n) \geq 0$. Thus player i obtains a positive payoff in some periods, that is, he does not seek contact with each opponent in these periods. Formally, there exists $t < T$ such that $L_i(g(a^t)) > 1$. Let a_i^t be the action played by player i in such a period. As in the proof of lemma 1, player i is able to deviate from M_i towards a $(T-1)$-state machine that simulates what plays M_i against M_{-i} in the first $T-1$ periods and then transits in stage T to the state playing action a_i^t. Using this altered strategy, player i must obtain in stage T a payoff $\pi_i(a_i^t, a_{-i}^T) > \pi_i(a^T)$ as he seeks less contacts and $\pi_i(a^T)$ is the worst payoff in the game. All other stage payoffs being identical, the deviating strategy yields player i a larger average payoff than M_i. This contradicts the fact that $(M_1, \ldots, M_n)$ is a RNE.

(iii) Consider any RNE $(M_1, \ldots, M_n)$ for which $g(a^{T-1}) = g(a^T) = g_0$ and for some periods $t \leq T-2$, $g(a^t) \neq g_0$. Let

$$t^* = \max_{t \leq T-2} \{t : g(a^t) \neq g_0\}$$

be the most remote period in which a nonempty network forms. By proposition 1, at least one player i is such that $d_i(a^{t^*}) > 0$. Suppose that i chooses to deviate from M_i towards a (t^*+1)-state machine that mimics M_i's behavior against machines M_{-i} up to period t^*-1, then plays action b_i in stage t^* and transits to a (t^*+1)th state playing action $a_{i,0}$ until the end of the game (see proof of lemma 1). Clearly, such a deviation yields player i a larger payoff, which implies that $(M_1, \ldots, M_n)$ is not a RNE. We conclude that two empty networks cannot form in the last two stages of a nonempty RNE. ■

These results lead to some conclusions. Result *(i)* has several interpretations. Firstly, the only minimal network that is likely to be connected is the last that forms. Secondly, there may be other connected networks in previous periods but this result implies that these networks must contain the last one. We may say that the formation of a connected network (if it occurs) has to be progressive. A connected network may form quickly in the process but it will be over-connected. If a connected network forms in a given period $t < T$ (of a RNE) and another one forms in the final period, the last network is more beneficial to all players and strictly more to one of them. Even a link formation process by a player generates an externality on the set of direct neighbors, this effect would be gradually internalized by some players. Thirdly, one may also interpret the first result in proposition 5 as the impossibility that the outcome of a RNE consists in a sequence of connected networks that extends as time goes by. This once again emphasizes that too much over-connected networks fail to form in a RNE.

Result *(ii)* shows that in the last network that forms, a player cannot create a direct link with all opponents.[7] For instance, the complete network and the star network are two such networks, and cannot form in the last stage of a RNE. Recall that point *(i)* does not prevent players from creating a connected network in stage T. By point *(ii)*, if players are all connected in the last network induced by a RNE, any of them avoids the burden to seek contact with all others. This shows how players learn to divide the task of connecting the network up among themselves. A consequence of this result is that the diameter of the network formed in the final stage must satisfy $L(g(a^T)) > 1$. This could be interpreted as the absence of an extreme small-world effect as observed by Milgram (1967). Notice that the star network is the only tree of diameter 2. Thus, by point *(ii)*, if $L(g(a^T)) = 2$ then $g(a^T)$ is not a tree. In words, a small-world effect (diameter 2) in stage T is possible only with inefficient networks (see section 4 for efficiency considerations).

Result *(iii)* displays that if the players have established relationships in the $T-2$ first periods, some old connections remain or new links form in at least one of the two last stages. In other words, if players create links in early periods, they must maintain some former links or create new links in at least one of the last two periods. For example, relationships between individuals in a connected population of agents never completely disappear with time. Consider a market represented by a network of firms. Links model competition between firms and the finite horizon of T periods indicates the lifespan of the product. A firm leaving the market is symbolized by an isolated vertex. By result *(iii)*, if some firms have competed in the market in some of the first $T-2$ years, then the market cannot become empty of firms in the final years.

In proposition 5, we provide necessary conditions that any RNE has to fulfill. In the next proposition we are going to state two sufficient structural properties for a given strategy profile to be a RNE.

[7]However, it is possible that a player obtains less than the minmax payoff in stage T.

Proposition 6 *Consider a strategy profile* $(M_1, \ldots, M_n)$ *with outcome* $(a^1, \ldots, a^T)$ *such that, for any* $i \in I$ *and any* $t \leq T$, $\pi_i(a^t) \geq 0$, *and*

$$\pi_i(a^T) \geq \max \left\{ d_i(a^{T-1}) - c \min_{a_{ij}^1, \ldots, a_{ij}^{T-1}} \sum_{j \neq i} a_{ij}^t ; \max_{k<T-1} d_i(a^k) - \sum_{t=k+1}^{T} \pi_i(a^t) \right\}. \quad (9)$$

If either,

(i) $g(a^t) \cap g(a^{t'}) = \emptyset$, *for any* $t, t' \in \{1, \ldots, T\}$ *or*

(ii) there is a permutation $p : \{1, \ldots, T-1\} \longrightarrow \{1, \ldots, T-1\}$ *such that* $g(a^T) \subseteq g(a^{p(1)}) \subseteq g(a^{p(2)}) \subseteq \cdots \subseteq g(a^{p(T-1)})$,

then $(M_1, \ldots, M_n)$ *is a RNE of* G^T.

Proof. Assume that for any $i \in I$ and any $t \leq T$, $\pi_i(a^t) \geq 0$. The proof has two parts. Firstly, we show that in both situations *(i)* and *(ii)*, a player has no incentive to deviate as in the proof of lemma 1.

(i) The assumptions that any action profile in the outcome is cost-efficient and that for any $t, t' \leq T$, $g(a^t) \cap g(a^{t'}) = \emptyset$ implies that for any $i \in I$, $N_i(g(a_i^t, a_{-i}^{t'})) = N_i(g(a_i^{t'}, a_{-i}^t)) = \{\emptyset\}$.[8] As player i may still intend to create some links, this implies that $\pi_i(a_i^t, a_{-i}^T) \leq 0$ and $\pi_i(a_i^{t'}, a_{-i}^T) \leq 0$. By assumption, we then have $\pi_i(a_i^t, a_{-i}^T) \leq \pi_i(a^T)$ and $\pi_i(a_i^{t'}, a_{-i}^T) \leq \pi_i(a^T)$. This means that there is no period $t < T$ such that player i benefits from using the automaton constructed in the proof of lemma 1.

(ii) The assumption that for any $t \leq T$, $g(a^T) \subseteq g(a^t)$ implies that $g(a^T) \cap g(a^t) = g(a^T)$. Thus, for any player $i \in I$, $g(a^T) = g(a_i^t, a_{-i}^T) \subseteq g(a^t)$. This relation can be rewritten as $N_i(g(a^T)) = N_i(g(a_i^t, a_{-i}^T)) \subseteq N_i(g(a^t))$. We also know that the cost to player i in a^t is not less than in a^T. Therefore, for any $i \in I$ and for any $t \leq T$, $\pi_i(a^T) \geq \pi_i(a_i^t, a_{-i}^T)$. This implies once again that there is no period $t < T$ such that player i benefits from using the automaton constructed in the proof of lemma 1.

Secondly, we prove that condition (9) guarantees that $(M_1, \ldots, M_n)$ is a RNE. Each machine M_i in $(M_1, \ldots, M_n)$ is assumed to include a punishment state playing $a_{i,0}$ to threaten any deviation by an opponent as the one in machine M^1 in the proof of proposition 3. It is not the interest of player i to deviate in stage T in both situations *(i)* and *(ii)*. Thus, two cases similar to the one in the proof of proposition 3 must be considered.

[8] As $m_i < T$, only $T-1$ totally different networks may form. We thus assume that the empty network g_0 forms in at least two periods. This does not contradict condition *(i)* since trivially $g_0 \cap g_0 = \{\emptyset\}$.

Case 1.
Player i can deviate by using a $(T-1)$-state machine that plays b_i in stage $T-1$ and $a_i = \arg\min_{t<T} c \sum_{j \neq i} a_{ij}^t$ in the last period (remember that player i cannot use a new state playing $a_{i,0}$ is stage T since $m_i < T$). Player i will not choose such an automaton if and only if

$$\begin{aligned} & \pi_i(a^{T-1}) + \pi_i(a^T) \geq \pi_i(b_i^{T-1}, a_{-i}^{T-1}) - c \min_{t<T} \sum_{j \neq i} a_{ij}^t \\ \iff \quad & \pi_i(a^T) \geq d_i(a^{T-1}) - c \min_{t<T} \sum_{j \neq i} a_{ij}^t. \end{aligned} \tag{10}$$

Case 2.
Player i can deviate in period $k < T-1$ by playing b_i and then using a (possibly) new state playing $a_{i,0}$ until the end of the game. It is not the interest of player i to behave like this if and only if

$$\sum_{t=k}^{T} \pi_i(a^t) \geq \pi_i(b_i^k, a_{-i}^k) \iff \pi_i(a^T) \geq d_i(a^k) - \sum_{t=k+1}^{T} \pi_i(a^t). \tag{11}$$

Condition (9) in proposition 6 results from the combination of (10) and (11). By condition (9) we know that these deviations are not profitable to player i. This concludes the proof. ■

In words, if the payoff that each player obtains in the last round is large enough, then two kinds of interesting structures are likely to emerge. These structures are antagonistic. In the first one, at each round a new static network is built. These networks have no common link with any network formed in previous periods. As a consequence, if any two players are direct neighbors in a given period, this is the first period for which they are direct neighbors and they will never be directly connected once again. In a sense, we can refer to such an equilibrium as one with forgotten neighbors. This may highlight that players prefer a variety of one-period direct neighbors than long term direct relationships. In economic relationships, such a pattern of behavior is quite common. In a trading market, some buyers prefer visiting a variety of sellers than establishing a long term relationship with a particular seller (the so-called searchers).

In the second one, networks share an identical subnetwork and the structure allows for a total order relationship among networks that form within a RNE. The sequence of static networks corresponding to the equilibrium may reveal a contraction phenomenon. At each new period, the network that forms may be more and more restricted. This is the case of many economic situations. Consider for example several firms competing in a new market. A link between two firms can represent the investment supported by both firms to differentiate their product from that of the other firm. As time goes by, least competitive firms are not strong enough to face competition. They either stop investing to differentiate their product (cuts links but preserve some) or leave the market (remain isolated). The competition network may retract progressively. Another interpretation is that more and more efficient networks may form within a RNE (see next section). The network should be k-connected in the initial period. The contraction process may lead to the formation of a tree network in the last stage. The total order may also exhibit an expansion phenomenon from the initial period to period $T-1$ and then a network contained in all others forms in the last round. If all networks in the sequence are connected, one can interpret such a phenomenon as $T-1$ periods of test before stage T in which agents choose the most valuable configuration.

In proposition 6 each player's payoff is assumed to be positive at each round. Despite the drastic restrictions on the intertemporal consistency between networks formed in any RNE, the next result shows that it is possible that any static network based on a cost-efficient action profile forms within a RNE. In the next proposition, we give a sufficient condition on the number of players for which any static network can appear in the outcome of a RNE.

Proposition 7 *Fix $n \geq 11$. There exists a RNE $(M_1, \ldots, M_n)$ whose outcome contains any network $g(a^*) \subseteq g_I$, $a^* \in A^{ce}$, at least once.*

Proof. Fix $n \geq 11$. We prove that any network $g(a^*) \subseteq g_I$, $a^* \in A^{ce}$, can form in the first period of a RNE. Denote by $g(l')$ the network that consists of the line network $g(l)$ (see the proof of proposition 4) except that player h is isolated. Let $g(l'')$ be the network isomorphic to $g(l')$ with player j being the isolated player.

We proceed in two steps. In a first step, we exhibit an n-tuple of automata $(M_1, \ldots, M_n)$ such that any network $g(a^*) \notin \{g(l'), g(l'')\}$ forms in the initial period and the (n-player) maxmin line network $g(l)$ forms in all remaining periods. Any deviation from M_i will release a definitive minmax punishment. The choice to form $g(l)$ in each period $t > 1$ has been already justified in the proof of proposition 4. In such a network the player who obtains the worst payoff is better than in any other network. Therefore, the condition on the number of players that prevents deviations is smaller if $g(l)$ forms from period 2 than if any other network forms.

However, by lemma 1, $g(l')$ and $g(l'')$ cannot form within the outcome of a RNE in which $g(l)$ forms in the mast stage. In fact, $\pi_i(l_i{}', l_{-i}) > \pi_i(l)$ for the neighbor in $g(l)$ of the isolated player h in $g(l')$. this is also true of $g(l'')$. As a consequence, in a second step, we deal with the cases $g(a^*) = g(l')$ and $g(a^*) = g(l'')$. We show that $g(l')$ and $g(l'')$ can be sustained as a RNN.

In a first step, consider any network $g(a^*) \notin \{g(l'), g(l'')\}$ and suppose that each player i chooses the following automaton:

M^4 :

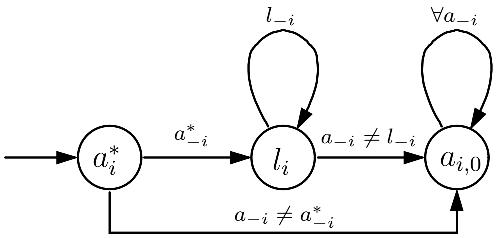


Figure 5.

As it is described in the first paragraph of the proof, the strategy profile $(M_1^4, \ldots, M_n^4)$ outputs network $g(a^*) \notin \{g(l'), g(l'')\}$ in period 1 and thereafter $T-1$ line networks $g(l)$. Player i obtains the average payoff

$$\tilde{\pi}_i(M_i^4, M_{-i}^4) = \frac{\pi_i(a^*) + (T-1)\pi_i(l)}{T}.$$

Any deviation induces a definitive minmax punishment. Thus, deviating in the first period yields player i the average payoff $\pi_i(b_i, a_{-i}^*)/T$. It is not the interest of player i to deviate in stage 1 if and only if

$$\frac{\pi_i(a^*) + (T-1)\pi_i(l)}{T} \geq \frac{\pi_i(b_i, a_{-i}^*)}{T} \iff (T-1)\pi_i(l) \geq d_i(a^*) \qquad (12)$$

Observe that $d_i(a)$ is maximal for a network $a \in A^{ce}$ in which player i seeks contact with each of the $n-1$ opponents whereas only one contact is needed to connect the network. Such a network being cost-efficient, player i keeps the network connected by removing $n-2$ superfluous links and saves $(n-2)c$. That is, for any action profile $a^* \in A^{ce}$,

$$\max_{a \in A^{ce}} d_i(a) = (n-2)c \geq d_i(a^*)$$

From the proof of proposition 4, we also know that the minimal payoff obtained by a player in $g(l)$ is $(n-1)v - 2c$. This implies that

$$\pi_i(l) \geq (n-1)v - 2c,$$

for any $i \in I$. Thus, condition (12) holds if

$$(T-1)((n-1)v - 2c) \geq (n-2)c \iff c \leq \frac{(n-1)(T-1)}{n+2T-4}v.$$

As $2v > c$ and $T > 3$, the reader can check that $n \geq 11$ is enough to guarantee that player i has no incentive to deviate. Next, consider a deviation in stage $t > 1$. The outcome $(a^*, l, \ldots, l)$ satisfies the necessary condition of lemma 1, that is, for any $i \in I$, there is no stage $k < T$ such that $\pi_i(a_i^k, l_{-i}^T) > \pi_i(l^T)$. In other words, players cannot benefit from deviating in stage T. Since we have also proved that $n \geq 11$ garantees that players will not deviate in stage 1, it remains to test deviations in stages $2, \dots, T-1$. The line network is formed in each of these stages. Thus, the condition that prevents deviations in these periods is identical to the one in proposition 3 for a RNN based on $g(l)$. To see this, recall that if a player has an incentive to deviate, then he does it late in the game because of the definitive punishment. Condition (3) in proposition 3 is satisfied for the RNN based on a line network when $n \geq 11$. Therefore, any network $g(a^*) \notin \{g(l'), g(l'')\}$ is likely to form in a RNE when $n \geq 11$.

In a second step, assume that each player i chooses the following two-state automaton:

M^5 :

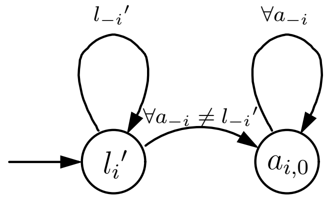


Figure 6.

The strategy profile $(M_1^5, \dots, M_n^5)$ leads to the formation of a repeated network based on $g(l')$. Once again, any deviation releases a definitive minmax punishment. This situation is a particular case of proposition 3. Therefore, it is enough to check that condition (3) is satisfied to show that $(M_1^5, \dots, M_n^5)$ is a RNN. As $n \geq 11$, remember that only the direct neighbors of the first and last players in the line component have an incentive to deviate from l'. Let i be one of these two players. We need

$$\begin{aligned} & d_i(l') \leq \min\left\{\frac{v}{2}\#N_i(g(b_i, l_{-i}')), 2\pi_i(l')\right\} \\ \iff \quad & c - v \leq \min\left\{(n-2)\frac{v}{2}, 2((n-1)v - 2c)\right\} \\ \iff \quad & c - v \leq (n-2)\frac{v}{2} \\ \iff \quad & n \geq \frac{2c}{v}, \end{aligned} \tag{13}$$

which always holds when $n \geq 11$. We conclude that network $g(l')$ can also form in an equilibrium of G^T. The networks $g(l'')$ and $g(l')$ being isomorphic, the same conclusion applies to $g(l'')$. ■

There is a contrast between this result and previous results of this section. On one hand, any static network can form if one considers just a particular period of the repeated game. On the other hand, the T-period outcomes of

RNE must satisfy restrictive conditions. In the initial period, players can set links, which form any network (proposition 7). But the formation of this first network then prevents players from creating some other networks in future periods (lemma 1 and proposition 5). For instance, if the network that forms in stage 1 is connected, then by proposition 5 this network cannot be a subnetwork of the network that forms in stage T. Thus, the initial network conditions the structure of the whole outcome of a RNE.

4 Efficiency

In this section, we characterize the set of efficient action profiles for both Bentham and Pareto criteria (see the appendix for the proofs). Next, we examine this question in the finitely repeated game and compare the results with those of the static case. Again, we consider two cases (players being perfectly rational or not) and we study the differences in the corresponding sets of efficient strategy profiles. Precisely, we give a condition on the duration of the game for which boundedly rational players can implement strictly less efficient strategy profiles than perfectly rational players. We denote by $W(a) = \sum_{i \in I} \pi_i(a)$ the welfare resulting from a.

Definition 5 *An action profile $a^* \in A$ is*

- *Bentham-efficient if $a^* \in \arg\max_{a \in A} W(a)$;*
- *Pareto-efficient if for any $i \in I$ and any $a \neq a^*$,*

$$[\pi_i(a) > \pi_i(a^*)] \Longrightarrow [\exists j \neq i | \pi_j(a) < \pi_j(a^*)].$$

We say that an action profile is efficient if it is efficient either in the sense of Bentham and/or in the sense of Pareto.

Proposition 8 *Suppose $n > 4$. An action profile $a^* \in A$ is efficient if and only if (i) it is cost-efficient and (ii) $g(a^*)$ is a tree.*

Thus in our model, the sets of Bentham and Pareto efficient action profiles do coincide. This is quite noteworthy. Remark that the star network with $n > 4$ players is efficient for both Bentham and Pareto criteria although the player in the center of the star obtains less than the minmax payoff. This shows that a static efficient network is not always sustained as a RNN. The efficiency of strategy profiles in the repeated game is defined with respect to the average payoff. Let A_E be the set of efficient action profiles and S_E be the set of efficient strategy profiles. We use the subscripts $_{BE}$ and $_{SE}$ for the Bentham and Pareto criteria. The set S_E is characterized in the following proposition.

Proposition 9 *A strategy profile s^* is efficient if and only if it induces an efficient action profile in each period. Formally,*

$$S_E = \{s \in S : a^t \in A_E, a^t \in h^{T+1}(s), t \leq T\}.$$

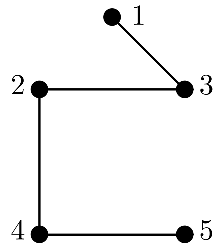
Proof. Firstly we prove that $S_{BE} = S_E$. A strategy profile s^* in which a Bentham-efficient action profile occurs in each period has a maximum average welfare $W(s^*)$ since the welfare is maximal is each period. Thus, the set of Bentham-efficient strategy profiles consists in all strategy profiles that induce T Bentham-efficient action profile, that is $S_{BE} = S_E$.

Secondly, we prove that any Pareto-efficient strategy profile induces a Pareto-efficient action profile in each period. We proceed by contradiction. Consider a Pareto-efficient strategy profile $s \in S_{PE}$ in which at least one non Pareto-efficient action profile a does occur. It is sufficient to suppose that there is only one such action profile, and that it occurs at round t. Thus, by definition, we know that there exists an action profile $p \in A$ such that, for any i, $\pi_i(p) \geq \pi_i(a)$, and $\pi_j(p) > \pi_j(a)$ for some j. Now consider the strategy profile s' that induces p at round t, and the same action profiles than s at the other rounds. Then we check easily than $\tilde{\pi}_i(s') \geq \tilde{\pi}_i(s)$, and $\tilde{\pi}_j(s') > \tilde{\pi}_j(s)$. This contradicts the Pareto-efficiency of s. By proposition 8, s' also induces a Bentham-efficient action profile in each period. We conclude by contradiction that $S_{PE} = S_{BE}$. ■

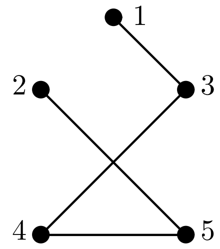
The results regarding the question of efficiency show that in our model Bentham-efficient and Pareto-efficient action profiles or strategy profiles have identical structures provided that the number of players is not too small. This correspondence between Bentham-efficiency and Pareto-efficiency is discussed in Jackson (2003) for the case of static network formation games.

A population of perfectly rational players can implement any efficient strategy profile in S_E. Now, let $\mathcal{M}_E$ denote the set of efficient strategy profiles of the machine game. A particular efficient strategy profile $s \in S_E$ may induce T different efficient networks. Thus, when players are assumed to use finite automata of limited size as in section 3, one can ask whether n automata with at most $T-1$ states may form T different efficient networks. In example 2, we show that combining actions played in the first $T-1$ stages may not allow for the creation of the Tth efficient network in the final period. We give a sequence of T line networks that cannot be implemented by automata with at most $T-1$ states. From this example, we will state a general proposition.

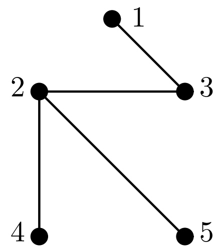
Example 2 Fix $N = \{1, 2, 3, 4, 5\}$, $T = 4$ and assume that players used strategies that can be implemented by automata with 2 or 3 states. Consider the sequences of four trees represented below and suppose that each action profile a^t, $t = 1, 2, 3, 4$, is cost-efficient.



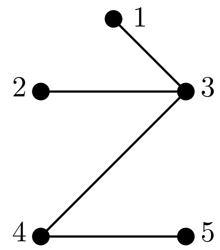
$g(a^1)$



$g(a^2)$



$g(a^3)$



$g(a^4)$

The strategy profile that induces the sequence of networks (a^1, a^2, a^3, a^4) is efficient. Remark that player 2 has a totally distinct set of neighbors in each period. To implement the four actions a_2^t, $t = 1, 2, 3, 4$, player 2 must use an automaton with 4 states which does not belong to his strategy set. It follows that there does not exist an automata profile with 2 or 3 states that implement (a^1, a^2, a^3, a^4). □

Next, we generalize the idea of example 2. Each player i has $2^{n-1} - 1$ different nonempty set of neighbors in the set of all networks with n players. Player i has also the same number of different nonempty set of neighbors in the set of all trees with n players. This remark leads to the following result.

Proposition 10 *Suppose* $2^{n-1} - 1 \geq T$. *Then* $\mathcal{M}_e \subsetneq S_e$.

Proof. Suppose $2^{n-1} - 1 \geq T$. We construct a sequence of cost-efficient action profiles $(a^1, \ldots, a^T)$ that induce T trees and such that for any two periods t and t', $n_i(g(a^t)) \neq n_i(g(a^{t'}))$. There exists such a sequence since $2^{n-1} - 1 \geq T$. It follows that $a_i^t \neq a_i^{t'}$ such that player i's automaton has to implement T different actions. This can be done only if player i's machine has at least T states. Such a strategy does not belong to $\mathcal{M}_i^{1T}$. Thus, there is no automata profile of size $1 < m_i < T$ for each $i \in N$ that induce $(a^1, \ldots, a^T)$. We conclude that $\mathcal{M}_e \subsetneq S_e$. ■

This result contrasts with those found on RNE. On one hand, the set of RNE is more important when players are boundedly rational (propositions 2 and 3). On the other hand, boundedly rational players can implement less efficient strategy profiles than perfectly rational players (proposition 10).

5 Conclusion

Within a finite-horizon repeated game framework we study the problem of (dynamic) network formation when the players are either perfectly rational, or boundedly rational in the sense of Neyman (1985), and by a restriction to a subset of action profiles. We prove that the set of RNE is reduced to the empty network when the agents are perfectly rational, while this set is much more elaborate when the complexity of their strategies is limited. Then we identify structural properties of RNNs and RNE. In the case of RNNs, we prove that each network that is sustained as a RNN cannot be too much over-connected, and that each player cannot bear a too important share of the cost needed to connect the network. This highlights a lack of robustness of architectures such as star networks. In the case of RNE, we prove that the networks induced in any period satisfy some structural properties. Within a RNE, players may prefer to set links with totally different partners at each round, or the networks may retract progressively. Bounded rationality has a noticeable influence both on the existence of (non trivial) equilibria and on the dynamics of network formation. Assuming a limited ability to implement link formation seems reasonable since it is consistent with well-known economic behaviors (searchers in a trading market for instance). Finally, we make some comparisons between the sets of (Bentham and Pareto) efficient strategy profiles. In this part, the nature of results is reversed. Under some condition between the duration of the game and the number of players, it is shown that more rational players will implement a larger number of efficient strategy profiles. One of the main assumptions of the present work is that consent is needed to form links. A possible extension to this paper would be to see what happens if this assumption is relaxed. In particular, the resulting networks may be directed with the consequence that information is only one-way flow. This is left for future research.

Appendix

Proof. (proposition 1) Firstly, we show that in any network $g(a) \neq g_0$, at least one player has an incentive to deviate. Secondly, we prove that $g_0 = g(a_0)$ is a Nash network.

Consider any network $g(a) \neq g_0$. Two cases must be studied:

1. Suppose that a is not cost-efficient, then $\exists i, j \in I$ such that $a_{ij} = 1 \neq a_{ji} = 0$. It is the interest of player i to choose action $a_i(j^-)$ that only

differs from a_i by $a_{ij} = 0$. Player i saves c while $g(a_i(j^-), a_{-i}) = g(a)$. He obtains $\pi_i(a_i(j^-), a_i) = \pi_i(a) + c$, which implies that $g(a)$ is not a Nash network.

2. Suppose that a is cost-efficient. If $g(a)$ is a cyclic network, then there are at least two players i and j such that $N_i(g(a) - ij) = N_i(g(a))$. Link ij is superfluous. We have $d_i^1(a) > 0$, which implies that $g(a)$ is not a Nash network. If $g(a)$ is an acyclic network, then there is a player i whose set of connections satisfies $\#N_i(g(a)) = 1$. Let $N_i(g(a)) = \{j\}$, then link ij is superfluous for player j. Therefore $d_j^2(a) > 0$, which implies $g(a)$ is not a Nash network.

The empty network $g_0 = g(a_0)$ is the unique network that does not fit with any of the two previous cases. In $g(a_0)$, there is no player i who has an incentive to deviate from $a_{i,0}$ since he cannot create links alone and would support a cost c for any such attempt. Then the cost-efficient empty network is the only Nash network of G. ■

Proof. (proposition 2) Consider network $g(a) \neq g_0$ and suppose $g(a^T) = g(a)$. By proposition 1 and a backward induction argument, there is at least one player say i who has interest in altering his action in the last period. Player i's opponents anticipate this behavior and also remove some links. As a consequence, the empty network necessarily forms in the last stage. A similar process leads to the formation of the empty network in all periods. ■

Proof. (proposition 8) The proof is divided in two parts, one for each criterion.

Bentham-efficiency

($\Longrightarrow$) Suppose $b \in A$ is Bentham-efficient. Firstly, b must be cost-efficient. Otherwise there are players $i, j \in I$ with $b_{ij} = 1 \neq b_{ji} = 0$ such that player i saves c if he plays $b_i(j^-) = 0$. Player i's altered action does not remove any link and maintains other players' payoffs. Remark that $g(b)$ must be acyclic, otherwise there is a player i in $g(b)$ who has some superfluous links or equivalently $d_i^1(b) > 0$. If i removes one such link, say with j, he obtains $\pi_i(b_i(j^-), b_{-i}) = \pi_i(b) + c$ and $\pi_h(b_i(j^-), b_{-i}) = \pi_h(b)$, $\forall h \neq i, j$ since $b_i(j^-)$ keeps unchanged all other players' connections (and payoffs).

Secondly, we have to show that $g(b)$ must be a tree. To see this, we prove that the welfare of a tree is larger than in any other type of network. In a tree, the n vertices must be connected by exactly $n-1$ links. The formation of a link costs c to two players. Thus, the total cost of a tree $g(b)$ is $(n-1)2c$. The value of $g(b)$ for each player is $(n-1)v$. The welfare of any tree $g(b)$

induced by a cost-efficient action profile b is [9]

$$W(b) = n(n-1)v - (n-1)2c = (n-1)(nv-2c). \tag{14}$$

Now consider any non connected network $g(a)$. By definition, $g(a)$ is splitted in $K > 1$ components, which are connected subnetworks. If $g(a)$ is candidate to be Bentham-efficient, then each subnetwork $g(a_k)$, $k \in \{1, \ldots, K\}$, satisfies condition 1 of the proposition, that is, $g(a)$ is a forest of K trees induced by cost-efficient action profiles. The component $g(a_k)$ has $\#I_k = n_k$ vertices (or players). The welfare of $g(a_k)$ is

$$W(a_k) = (n_k - 1)(n_k v - 2c),$$

and the total welfare of a is

$$W(a) = \sum_{k=1}^{K} (n_k - 1)(n_k v - 2c).$$

By definition of I_k,

$$nv - 2c > n_k v - 2c \Longrightarrow \sum_{k=1}^{K} (n_k - 1)(nv - 2c) > \sum_{k=1}^{K} (n_k - 1)(n_k v - 2c).$$

Furthermore, $\sum_{k=1}^{K} n_k - 1 = n - 1$ implies that

$$(n-1)(nv-2c) > \sum_{k=1}^{K} (n_k - 1)(nv - 2c),$$

and we conclude that $W(b) > W(a)$.

($\Longleftarrow$) Suppose b satisfies the two conditions listed in the statement of proposition 8. By the previous calculation, the welfare in b is larger than in any other action profile.

Pareto-efficiency

($\Longleftarrow$) By the first part of the proof, any tree induced by a cost-efficient action profile is Bentham-efficient. And it follows from definition 5 that any Bentham-efficient network is Pareto-efficient. Therefore, any cost-efficient action profile that induces a tree is a Pareto-efficient.

[9] Such a network ranges from the n-player line network to the n-player star network. These two networks exhibit extreme situations according to their diameter. A line network has a diameter of $n-1$, the largest diameter among trees, while a star network has a diameter of 2, the smallest diameter among trees.

($\Longrightarrow$) Firstly, any Pareto-efficient action profile must be cost-efficient for the same reason than in the first part of the proof. Secondly, to show that any Pareto-efficient action profile induces a tree, we argue by contradiction. Consider any Pareto-efficient action profile p for which $g(p)$ is not a tree. Note that $g(p)$ cannot be k-connected, $k > 1$, since the existence of some superfluous links would contradict the fact that p be Pareto-efficient. Then, suppose that the Pareto-efficient action profile p induces a non connected network. We group all possibilities in three cases according to the structure of $g(p)$:

Case 1.
Network $g(p)$ consists in at least 2 connected components with at least 2 players. Each component must be a tree to avoid cycles (and superfluous links). Let K be the total number of components in the forest $g(p)$. Construct the network $g(p')$ that consists in connecting the K components all together with the creation of $K-1$ links (see for instance the first part of the proof). Since $2v > c$, it is easy to check that the resulting network yields all players a larger payoff than $g(p)$. This proves that $g(p)$ cannot be Pareto-efficient.

Case 2.
Network $g(p)$ consists in a connected component and $k > 1$ isolated players. The component must be a tree to avoid cycles. The method described in case 1 is still beneficial to all players whenever at least 2 isolated players can be linked to the main tree component.

Case 3.
Network $g(p)$ consists in a connected component and a single isolated player. The component must also be a tree to avoid cycles. If the connected component includes all but one player denoted h, it follows that the creation of link ih with any player i in the component is beneficial to all players except i, who loses $v - c$. Nonetheless, it is possible to connect player h to all other opponents in a way that increases everyone's payoff. Let i be a player such that $\#n_i(g(p)) = 1$. Player i must exist since the connected component is a tree. Precisely, let $n_i(g(p)) = \{j\}$. Construct the network $g(p') = g(p) - ij + hj + hi$. The reader can check that all players except h have the same number of direct neighbors in $g(p')$ than in $g(p)$ and benefit from the additional connection with player h. Player h creates two links but is connected to at least 4 players. Thus, his payoff is larger than that of a isolated player. Therefore all players obtain a larger payoff, which implies that p cannot be Pareto-efficient.

Thus, we conclude by contradiction that any Pareto-efficient action profile induces a tree. ■

References

[1] Abreu Dilip, Rubinstein Ariel (1988), "The Structure of Nash Equilibrium in Repeated Games with Finite Automata", *Econometrica* **56**:1259-1282.

[2] Bala Venkatesh, Goyal Sanjeev (2000), "A Non Cooperative Model of Network Formation", *Econometrica* **68**:1181-1229.

[3] Bolton Patrick, Dewatripont Mathias (1994), "The Firm as a Communication Network", *Quarterly Journal of Economics* **109**:809-839.

[4] Calvó-Armengol Antoni (2003), "Stable and Efficient Bargaining Networks", *Review of Economic Design*, **71**:411-428.

[5] Calvó-Armengol Antoni, Jackson Matthew O. (2004), "The Effects of Social Networks on Employment and Inequality", *American Economic Review*, **94**:426-454.

[6] Currarini Sergio, Morelli Massimo (2000), "Network Formation with Sequential Demands", *Review of Economic Design*, **5**:229-249.

[7] Dutta Bhaskar, Ghosal Sayantan, Ray Debraj (2005), "Farsighted Network Formation", *Journal of Economic Theory*, **122**:143-164.

[8] Ellison Glenn (1993), "Learning, Local Interaction and Coordination", *Econometrica*, **61**:1047-1071.

[9] Ellison Glenn, Fudenberg Drew (1995), "Word of Mouth Communication and Social Learning", *Quarterly Journal of Economics* **110**:93-125.

[10] Gilles Robert P., Sarangi Sudipta (2004), "Social Network Formation with Consent", *Working Paper*, Department of Economics, Virginia Tech and Department of Economics, Louisiana State University.

[11] Goyal Sanjeev, Vega-Redondo Fernando (2005), "Network Formation and Social Coordination", *Games and Economic Behavior* **50**:178-207.

[12] Jackson Matthew O. (2003), "The Stability and Efficiency of Economic and Social Networks", in *Advances in Economic Design*, edited by Murat Sertel and Semih Koray, Springer-Verlag, Heidelberg.

[13] Jackson Matthew O., Watts Alison (2002), "The Evolution of Social and Economic Networks", *Journal of Economic Theory* **106**:265-295.

[14] Jackson Matthew O., Wolinsky Asher (1996), "A Strategic Model of Social and Economic Networks", *Journal of Economic theory* **71**:44-74.

[15] Milgram Stanley (1967), "The Small World Problem", *Psychology Today*, **2**:60-67.

[16] Neyman Abraham (1985), "Bounded Complexity Justifies the Cooperation in the Finitely Repeated Prisoner's Dilemma", *Economics Letters* **19**:227-229.

[17] Piccione Michele, Rubinstein Ariel (1993), "Finite Automata Play a Repeated Extensive Game", *Journal of Economic Theory*, **61**:160-168.

[18] Simon Herbert A. (1955), "A Behavioral Model of Rational Choice", *Quarterly Journal of Economics*, **69**:99-118.

[19] Watts Alison (2001), "A Dynamic Model of Network Formation", *Games and Economic Behavior* **34**:331-341.

[20] Watts Alison (2002), "Non-Myopic Formation of Circle Networks", *Economic Letters* **74**:277-282.