Modelling the Crossroad Traffic Flow and Adaptive Traffic Light Control

Julia Frolova[1] and Victor Korobitsin[1]

Omsk State University, Computer Science Department,
55A, Mira pr., 644077 Omsk, Russia
frolova@univer.omsk.su, korobits@univer.omsk.su

Abstract. The paper considers the problem of optimal control of traffic light. It begins with an investigation of the traffic flow on crossroad. The influence of traffic lights on congestion is shown. Proposed simulation tool gives the effective control system of crossing. The optimal time of green lights on the crossing is derived.

1 Introduction

The main problem of city life is traffic congestion. The present-day traffic networks are unable to efficiently handle the daily movements of traffic through urban areas. The primary focus should be to improve traffic flow without changing the layout or structure of the existing roadways. The inefficient control of traffic system decreases the economic level of urban traffic networks.

Current methods for enabling traffic to flow through intersections include building overpasses and installing traffic lights. However, the former is very expensive and forbids turning, while the latter can be quite inefficient, often requiring cars to remain stopped even when no cars are present on the intersecting road. In the paper the method for calculating the optimal value of green light traffic light is suggested.

The remainder of this paper is organized as follows. Section 2 describes the multi-agent model of moving the vehicles on crossroad. The results of simulation of vehicle dynamics on same lane of crossroad are given in Section 3. The result of calculating the outflow on the lane is presented in Section 4. Section 5 derives the optimal value of green light signal on crossroad with different inflow on lanes.

2 Description of Model of Moving the Vehicles on Crossroad

2.1 Dynamic Model of Moving the Vehicle

We suppose that moving the vehicle will obey the law of uniformly accelerated motion of solid. There are three coordinates x, y, φ for description of vehicle position on plane. The coordinates are functions of time t. We calculate the vehicle coordinates at discrete moments of time step-interval Δt:

$$\begin{aligned}
&x(t+\Delta t) = x(t) + r_x(t+\Delta t)\cos\varphi(t) + r_y(t+\Delta t)\sin\varphi(t),\\
&y(t+\Delta t) = y(t) + r_x(t+\Delta t)\sin\varphi(t) - r_y(t+\Delta t)\cos\varphi(t),\\
&\varphi(t+\Delta t) = \varphi(t) + r_\varphi(t+\Delta t),\\
&x(0) = x_0,\\
&y(0) = y_0,\\
&\varphi(0) = \varphi_0,
\end{aligned}$$

where x_0, y_0, φ_0 are initial coordinates of vehicle, specifyed according to entry lane; $r_x(t), r_y(t), r_\varphi(t)$ are the functions for defining moving the vehicle in self co-ordinates, they are calculated as:

$$\begin{aligned}
r_x(t+\Delta t) &= v(t+\Delta t)\cdot(\cos\alpha(t) + \tfrac{W}{2L}\sin\alpha(t))\Delta t,\\
r_y(t+\Delta t) &= \tfrac{1}{2}v(t+\Delta t)\cdot\sin\alpha(t)\cdot\Delta t,\\
r_\varphi(t+\Delta t) &= \tfrac{1}{L}v(t+\Delta t)\cdot\sin\alpha(t)\cdot\Delta t.
\end{aligned}$$

The function of velocity $v(t)$ changes according to signals of control light P, k_D, k_A:

$$v(t+\Delta t) = \begin{cases} v(t) - D\cdot k_D(t)\Delta t, & \text{if } P(t) = 1,\\ v(t), & \text{if } P(t) = 2,\\ v(t) + A\cdot k_A(t)\Delta t, & \text{if } P(t) = 3. \end{cases}$$

2.2 Description of Driver Behaviour

The algorithm of driver behaviour on crossroad includes three blocks: the following for vehicles, observation for traffic lights, motion on route. Each component forms the control signal, obtained from external influence upon system. As a result of three controlling signals is formed one, integrating all signals in itself. It is driver control signal that is transmitted for vehicle.

The algorithm of following for vehicles. Here we describe the rule for driver behaviour. It allows to save distances to previous vehicle.

1. We calculate maximum radius of view field on roadway by driver:

   $$S_r = t_r \cdot v(t).$$

   It shows the change of vehicle position in a driver reaction time t_r if the vehicle will continue motion at the velocity $v(t)$.
2. We scan all points of crossroad on vehicle route from current point to point moved on distance S_r.
3. If other vehicle is found in some point then we calculate the distance d_a to it and we define its velocity v_a. Then we calculate the force to brake pedal. It is necessary for prevention of breakdown:

   $$f_1 = \begin{cases} 0,01, & \text{if } F_1 < 0,01,\\ F_1, & \text{if } 0,01 \le F_1 < 1,\\ 1, & \text{if } F_1 \ge 1, \end{cases} \quad \text{where } F_1 = \frac{(v(t) - v_a)^2}{(d_a - L)\cdot D}.$$

The subsystem returns value $P_1 = 1$ (the brake pedal) and force f_1 in case when at least one vehicle was found, otherwise transmites control signal that changes are none.

The algorithm of observation for traffic lights. Here we describe the second rule for driver behaviour. It allows to keep the light signals on crossroad. This rule runs for the time being the vehicles did not cross stop line.

If current light signal is red for this vehicle, then we define force to brake pedal. We suppose to the vehicle has time to braking as far as stop line.

$$f_2 = \begin{cases} 0,01, & \text{if } F_2 < 0,01, \\ F_2, & \text{if } 0,01 \le F_2 < 1, \\ 1, & \text{if } F_2 \ge 1, \end{cases} \quad \text{where } F_2 = \frac{v(t)^2}{r \cdot D},$$

here r is distance from vehicle to stop line.

The subsystem returns value $P_2 = 1$ and force f_2 in case when red light signal, otherwise transmites control signal that changes are none.

The algorithm of motion on route. The vehicle must pass on defined route on crossroad. (see.p. 2.3).

According to this route the driver controls the vehicle so as following conditions will satisfy: vehicle has near to key points, vehicle velocity must be near to key value. We estimate the difference $v_i - v(t)$ for controlling the speed mode, where v_i is key value of velocity.

If $v_i - v(t) > 0$, then

$$P_3 = 3,\ f_3 = \frac{(v_i - v(t))^2}{r \cdot A},$$

else if $v_i - v(t) < 0$, then

$$P_3 = 1,\ f_3 = \frac{(v(t) - v_i)^2}{r \cdot D},$$

in case $v_i - v(t) = 0$:

$$P_3 = 2, f_3 = 0,$$

here r is distance from vehicle to key point

$$r = \sqrt{(x(t) - x_i)^2 + (y(t) - y_i)^2}.$$

The angle of turning the steering wheel is calculated by:

$$\alpha_3 = \arctan \frac{2R_x}{R_y - \frac{W}{L} R_x},$$

where

$$R_x = -(x_i - x(t)) \sin \alpha(t) + (y_i - y(t)) \cos \alpha(t),$$
$$R_y = (x_i - x(t)) \cos \alpha(t) + (y_i - y(t)) \sin \alpha(t),$$

here x_i, y_i are coordinates of current control point.

We suppose that vehicle achieved control point if distance to it less defined parameter ε_R. Then we change control point and increase variable i by 1.

The choice of main mode of control. Stated above the algorithms form the control signals, united in one signal. This signal is driver decision. Main signal is signal required maximum braking among all. But if deceleration signals is not, that is chosen the acceleration signal. The control parameters P^*, k_D^*, k_A^*, α^* are formed according to the following algorithm:

1. Set $P^* = 2, k_D^* = 0, k_A^* = 0, \alpha^* = 0$.
2. If $P_1 = 1$, then $P^* = 1, k_D^* = f_1$.
3. If $P_2 = 1$ and $f_2 > k_D^*$, then $P^* = 1, k_D^* = f_2$.
4. If $P_3 = 1$ and $f_3 > k_D^*$, then $P^* = 1, k_D^* = f_3$.
5. If $P_3 = 3$ and $P^* \neq 1$, then $P^* = 3, k_A^* = f_3$.
6. Set $\alpha^* = \alpha_3$.

Let the control functions $P(t), k_D(t), k_A(t), \alpha(t)$ equal values $P^*, k_D^*, k_A^*, \alpha^*$:

$$P(t) = P^*, k_D(t + \Delta t) = k_D^*,\ k_A(t + \Delta t) = k_A^*,\ \alpha(t + \Delta t) = \alpha^*.$$

2.3 Forming the Way of Vehicle

When agent-vehicle is created, it has prefixed way, which it pass on crossroad. This way is specifyed four key points. On the figure 2 is revealed as an example such ways. The point “0” is the place of vehicle appearance. The point “1” is the place of crossing the way with traffic light stop line. The point “2” is the place on lane, where the vehicle turn out after crossing. The point “3” is the place, where vehicle must abandon the crossroad.

Every i point ($i = 0, 1, 2, 3$) is described four parameters: x_i, y_i, α_i, v_i. These parameters indicate the desired position of vehicle and its velocity at passing of key point. The coordinates of key points are specifyed according to requireded way. The velocity is restricted by physical parameters of crossroad (the radius of turning, condition of road covering and other factors). In this case we install parameters according to following rules:

1. In point “0” is assigned initial velocity $v_0 = 5$ [m/s].
2. In points “1” and “2” velocity is chosen according type of turning:
   a) forward: $v_i = 15$ [m/s];
   b) to the left: $v_i = 8$ [m/s];
   c) to the right: $v_i = 5$ [m/s].
3. In point “3” is assigned the maximum velocity $v_3 = 20$ [m/s].

3 Geometric Features of Crossroad

The first geometric test shows the velocity rate dynamics. A close look at figure 2 reveals this process. The influence of control lights and interaction of vehicles doesn’t apply here. The three curves demonstrate the dynamics of vehicles that are moving in three directions: forward, to the left and right sides.

The averange values of crossroad parameters were determined according to directions:

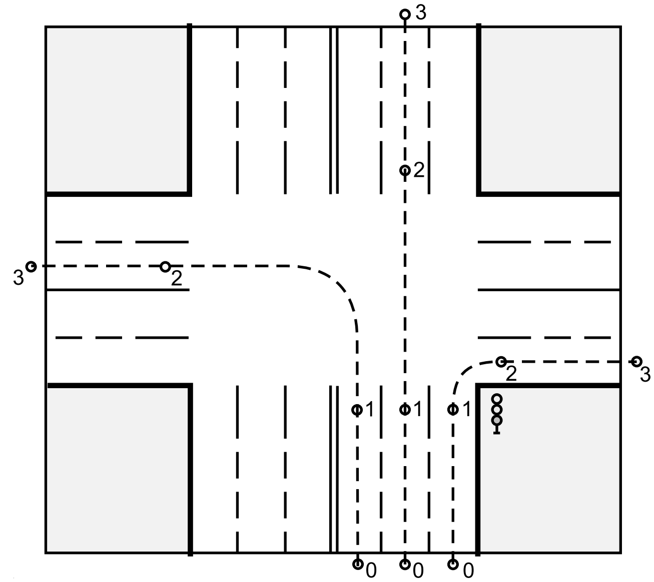

Fig. 1. Key points of way of vehicle on crossroad.

- forward:
  $t = 7.00$ s, $s = 80.90$ m, $v_{max} = 18.50$ m/s, $v_{mid} = 11.51$ m/s.
- to the left:
  $t = 8.25$ s, $s = 82.83$ m, $v_{max} = 16.58$ m/s, $v_{mid} = 10.01$ m/s.
- to the right:
  $t = 9.85$ s, $s = 70.02$ m, $v_{max} = 13.59$ m/s, $v_{mid} = 7.10$ m/s.

Here t is a time for passing the crossroad, s is full distance, v_{max} is maximal velocity, v_{mid} is averange velocity.

The second geometric test shows the velocity rate dynamics under influence of control lights (figure 3). The three vehicles were appeared on the cross in different moments (the first on 0 s, the second on 20.70 s, the third on 30.05 s) and on different lanes. Every vehicle reached the stop line and stopped than it is waiting the green light signal. After that all vehicles started in moment at 37.50 s.

The vehicle motion dynamics on the same lane is demonstrated on the figure 4. The curves on the figure show the position of vehicle on crossing that is changing in time. The vehicles are appearing on crossing in various moment that is marked by beginning the curve on the time axis. All curves finished on the distance 80 m. This point corresponds the boundary achievement. Two characteristic cases of data (A and B) are choosing on the figure.

The case “A” correspond to moment $t = 25.60$ seconds. At this time the light signal switch to green light. The distances between vehicles is demonstrated on figure at moment “A”. On this example the distances between center of vehicles

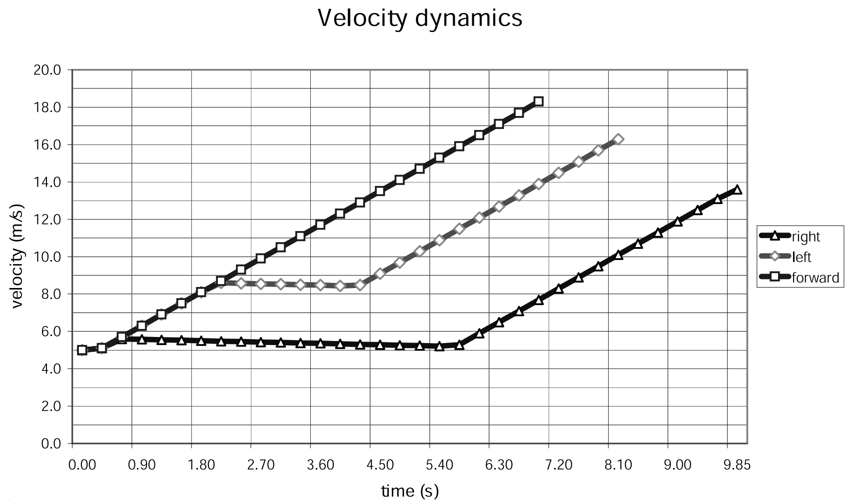

Fig. 2. The velocity rate dynamics in unlimited mode.

are: 4.31; 4.26; 6.39; 5.60 meters. Therefore the averange distance is from 0.26 to 2.39 meters.

The case "B" shows the time interval for replacement of vehicle by follower. The measuring of time was on entry point for crossing. On this example the replacement intervals are: 2.30; 0.90; 0.95; 0.65; 0.70 seconds. These values demonstrates the rate of overcoming the crossing. We can see that the first value (2.30 s) indicates the acceleration process and other the replacement time for followers. Theses values are from 0.65 to 0.95 seconds. The averange replacement time τ is 0.8 seconds.

4 Calculation of Outflow

The modelling results permit to estimate the capacity of crossroad. For estimating we must to calculate the maximum amount N of vehicles that are overcome the crossroad during the green light signal. This value we can derive from condition:

$$\sum_{i=1}^{N} \tau_i \leq T_s,$$

here τ_i is the replacement time for i vehicle, T_s is green light time (in example, $T_s = 15$ seconds). Take to account the averange replacement time calculated in previous paragraph we get:

$$N = \frac{T_s - \tau_1}{\tau} = \frac{15 - 2.3}{0.8} = 15.875.$$

This value shows that the maxumum amount of vehicles overcomed the crossing is less than 16 for green light time $T_s = 15$ seconds. For our example, where

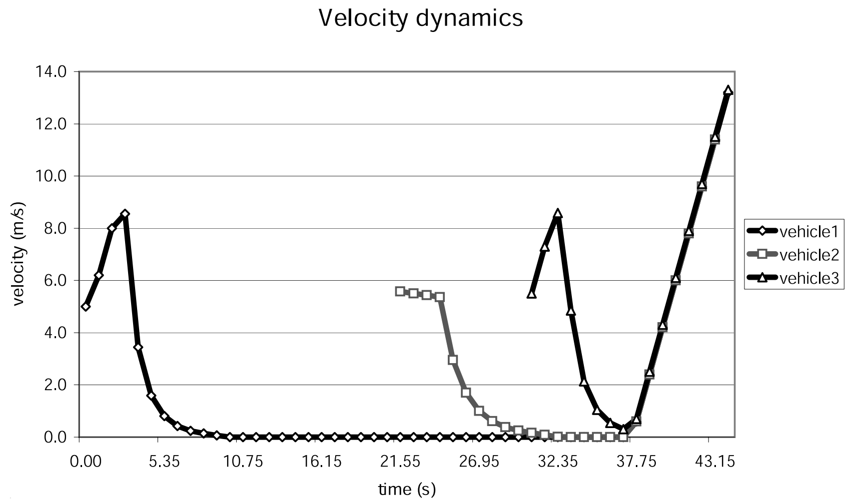

Fig. 3. The velocity rate dynamics under light control.

number of directions $N_s = 4$, time of yellow signal $T_y = 3$, we get the outflow P_{out} of crossing:

$$P_{out} = \frac{60 \cdot N}{N_s(T_s + T_y)} = \frac{60 \cdot (T_s - \tau_1)}{\tau N_s(T_s + T_y)}. \tag{1}$$

For given parametes of model we get

$$P_{out} = \frac{60 \cdot (15 - 2.3)}{0.8 \cdot 4 \cdot (15 + 3)} = 13.23 \text{ [vehicle/minute]}.$$

The graph of function $P_{out}(T_s)$ is showed on figure 5. We see that this function is limit on top. The maximum value has achieved with increasing T_s.

5 Deriving the Optimal Value of Green Light Signal

The inflow of vehicles is determined by parameters λ from the formula:

$$P_{in} = \frac{60}{\lambda} \text{ [vehicle/minute]}.$$

In the case $P_{in} < P_{out}$ all entried vehicles will leave the crossing, but in case $P_{in} > P_{out}$ the traffic jam is appeared. So when the parameter

$$\lambda < \frac{N_s(T_s + T_y)}{N}$$

the queue on the crossing is arrised. In our example the parameter λ must be more than 4.54 so the queue do not grow. The value $\lambda = 4.54$ according to inflow value $P_{in} = 13.23$ [vehicle/minute].

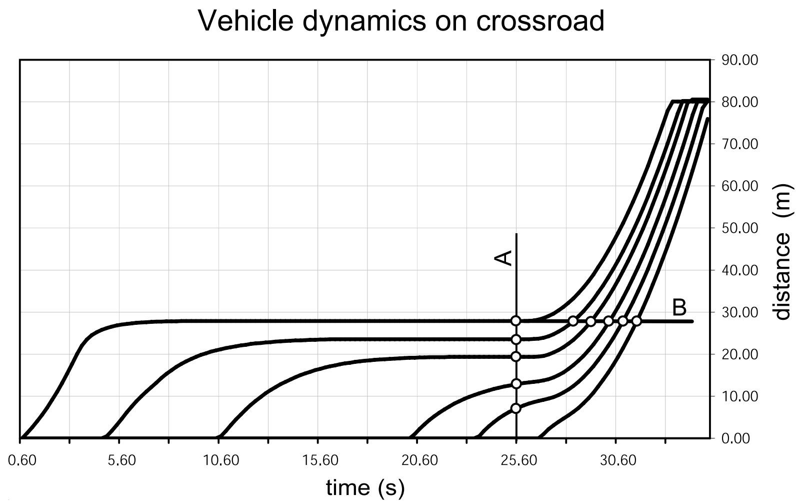

Fig. 4. The vehicle moving dynamics on the same lane under light control.

In case the inflows are different and they equal to $P_{in}^1, P_{in}^2, ..., P_{in}^{N_s}$ [vehicle/minute] we can derive optimal value of green light signal T_s^i for every flow if we suppose $P_{in}^i = P_{out}^i$. Putting the expressions for P_{in}^i and T_s^i in one for P_{out} we has got the system of equations relative to unknown values T_s^i:

$$P_{in}^i = \frac{60 \cdot (T_s^i - \tau_1)}{\tau\left(\sum\limits_{j=1}^{N_s} T_s^j + N_s T_y\right)}, \quad i = 1, 2, ..., N_s.$$

Convert the system to:

$$\left(\frac{60}{\tau P_{in}^i} - 1\right) T_s^i - \sum_{j \neq i} T_s^j = \frac{60 \cdot \tau_1}{\tau P_{in}^i} + N_s T_y, \quad i = 1, 2, ..., N_s. \qquad (2)$$

The system (2) is a linear and the solution is easy determined by Gauss's method for example.

We give the example of solution. For defined inflow: $P_{in}^1 = 10$, $P_{in}^2 = 15$, $P_{in}^3 = 12$, $P_{in}^4 = 8$ [vehicle/minute] we get the optimal values for green light signal time: $T_s^1 = 9.37$, $T_s^2 = 12.90$, $T_s^3 = 10.78$, $T_s^4 = 7.95$ [seconds].

6 Conclusions

The designed model of vehicle motion on crossroad and the simulation result allow to make the following conclusions:

1. The vehicle replacement time τ_i is defined by geometric features of crossroad, parameters of vehicles and behaviour of drivers. We showed that the parameters τ_i may be replaced by average value τ for any $i = 2, 3, ...$.

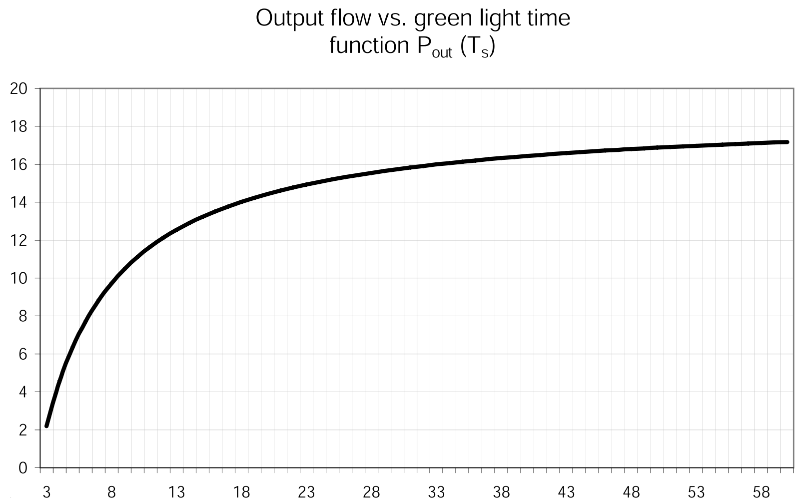

Fig. 5. The function of maximum outflow value for crossroad with identical flows.

2. The outflow P_{out} is limited either by value of inflow P_{in} or by function (1) depended on green light signal time T_s.
3. For different inflows the optimal values for green light signal time is found from system of linear equations (2).

References

1. Button KJ, Hensher DA (2001) Handbook of Transport Systems and Traffic Control. Elsevier Science
2. Chowdhury MA, Sadek AW (2003) Fundamentals of Intelligent Transportation Systems Planning. Artech House Publishers, Norwood
3. Colombo RM (2002) A 2× 2 hyperbolic traffic flow model. Mathematical and Computer Modelling 35: 683–688
4. Griffiths JD (1998) Mathematics in Transport Planning and Control. Pergamon Press Inc., New York
5. Helbing D (2001) Traffic and Related Self-driven Many-particle Systems. Reviews of Modern Physics 73:1067–1141
6. Helbing D, Hennecke A, Shvetsov V, Treiber M (2002) Micro- and macro-simulation of freeway traffic. Mathematical and Computer Modelling 35: 517–547
7. Roozemond DA (2001) Using Intelligent Agents for Pro-active, Real-time Urban Intersection Control. European Journal of Operational Research 131: 293–301
8. van Katwijk R, van Koningsbruggen P (2002) Coordination of traffic management instruments using agent technology. Transportation Research 10: 455–471