Fractal Analysis of Different Eastern and Western Musical Instruments

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Abstract:
In this paper, we attempt musical analysis by measuring fractal dimension (D) of musical pieces played by several musical instruments. We collected solo performances of popular instruments of Western and Easter origin as samples. We attempted usual spectral analysis of the selected clips to observe peaks of fundamental and harmonics in frequency regime. After appropriate processing, we converted them into time series data sets and computed their fractal dimension. Based on our results we conclude musical sounds may have higher Ds
Key Words: music, spectral analysis, time series analysis, fractal dimension

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1. Introduction

Musical analysis is a complex task. The basic question is: Is it possible to quantify good music? Music creates feelings to the listener that cannot be easily quantified. So in order to answer the question, one can try to find some common property of only those musical pieces which are already established as 'good music'- for example, performances of well known musicians. Actually, there is a fundamental relationship between our perception of beauty in melody and musical form and the presence of fractal qualities in music. Aspects of certain theories about the origins and fundamental structures of melodies suggest that much—perhaps all—beautiful music is, in some essential sense, fractal in its melodic material and internal self-similarity. [1]

Mathematical analysis of music is not new though there is no standard method. In 1975, Voss and Clarke formulated a provocative statement that may be summarized – in simplified form – as follows: Music is 1/*f*-noise. Their conclusion [2,3] is based on spectral time series analysis of recorded music. Again, for analysis of these time series, there is no standard tools. Given the complex nature of the data, here we try to apply nonlinear tools. The facts that music exhibits 1/f-power spectra at low frequencies and music possessing self-similarity allow us to consider music as a time series and analyze the fractal dimension of a particular piece of music. Bigerelle and Iost [4] found the global fractal dimension (D) to be an invariant for different types of music. In another work [5], D in the music of Mozart and Bach was calculated. Hsu and Hsu [6] discussed application of D to music in detail and in a work of Bach, found D to be 2.418. Different techniques proposed for automatic classification of musical instruments have been reviewed in [7]. In another work, the Zipf-Mandelbrot law was applied to musical pieces to computationally identify and emphasize aesthetic aspects of music. Zipf observed that phenomena generated by complex social or natural systems (such as human language and music), tend to follow a statistically predictable structure. [8].

In this paper, we shall attempt fractal analysis of musical pieces played by different musical instruments. The study is in line with our previous work [9] where we calculated D of different types of Indian songs. We found that for classical songs, value of D is much higher than that for light songs, while it for semi-classical songs lies in between the two. In this work, we propose to incorporate the same idea of fractal analysis to musical instruments- rather than for vocal performances. Here we have selected solo performance of the following Eastern and Western musical instruments: tabla, sitar, sarod, tanpura, saxophone, santoor, piano, organ, guitar, flute, and violin. In the previous case, it was known that, for example, classical songs are sung following a set of rules which essentially has many ups and downs in the frequency regime- so classical songs logically had higher D. Compared to that, while working with musical instruments, we have no prior knowledge when we collect samples of different musical instruments for proposed analyses. Also there is no scope of such knowledge about the type of performance because the same instrument can be used to play a classical or a light music. Still we want to investigate the music produced by different instruments because they all have different nature of production of sound that is invariant to what it plays.

For some of the perhaps lesser-known Eastern instruments (first four of the list), we have included a short note at the end. The musical pieces are different and have been chosen randomly. Also the instruments are different in the physical way they produce sound- that is some have strings, some are percussion types while some others are wind-driven. For a detailed discussion about different instruments, there are many excellent resources- for example one given in reference [10].

We collected 11 audio clips of music in MP3 format - each one played by different instruments mentioned above as described in Sec 2. We then made spectral analysis of each of them and calculated D of excerpts in Sec 3. We present the results derived from examples of different types of melodic lines in section 4 where some figures will be drawn for frequency analyses. Based on results obtained, some conclusion will be made in section 5.

But question remains as stated at the beginning. How much can music be understood in a quantitative manner? The sound wave of a musical performance is mainly determined by the notes that are played, the instruments, the way the instruments are played, and specific acoustic conditions (room, microphone, etc.). Efforts are made to analyze the sound wave. Acoustics is, however, only one aspect of music. Music is not just an arbitrary collection of sounds, but rather “organized sound,” or, as the German philosopher and mathematician Leibniz (1646-1716) put it: “Music is the arithmetics of the soul.” The fractal character is only one of many aspects that define a composition. The human mind may use one or more models of perception in order to determine whether a given melody or musical structure is ugly or beautiful. [11] Accordingly, we don’t attempt to make any final qualitative comment- like good or bad music - about the musical pieces that will be under study in this paper.

2. Data

We have selected 11 solo performances, each one played by different instrument. The original soundtracks are in MP3 format. We have extracted a roughly 14s clip from each one. The musical pieces are being solo in nature, so only sound recorded is of that particular instrument; there is no other accompanying sound. Also, the instruments are so chosen that they differ in physical process of production of sound (that is, wind, string or percussion type) and also their region of origin/popularity (that is, both Eastern and Western).

We have used the Sigview™ Spectrum analyzer to convert the wave files from time domain to frequency domain. This lets us to better understand the fundamentals, harmonics and overtones in the piece. We have extracted the waveforms of the musical excerpts and converted them to ASCII data- that is in text form. So finally we have a data file for each musical piece. While recording and converting audio files on a computer, we used the following parameters: single channel, 8 bit and 11 kbps sampling rate for recording and playback purpose in order to keep the data file size small (still each file has nearly 120000 data points.). These data files are fed again to computer program that playbacks the original soundtracks and a real time calculation of D is done simultaneously.

3. Analysis of the data

For the repetition of frequency in the time scale may lead to some self similarity in the time series plot. To investigate this property, standard statistical tools are not sufficient. A more appropriate tool seems to be the fractal dimension.

3.1 Fractal Dimension (D)

D is a measure of the extent to which trajectories on the attractor fill a region in the phase space; a strange attractor has a noninteger dimension. There are at least five well-established different definitions of D, although their interrelations are by no means completely understood [12]. If we try to cover, suppose, a line segment with squares of some finite side, say R, then let N be the number of squares of that size is required to cover the set. Now let us make the square small enough so that the curve (whose D is being estimated) is approximated well. If we plot in phase space, each point represents a state of the system. In the limiting case where scaling R makes each square to contain approximately a single point, then N represents number of states. So, for one dimensional objects with finite set, in the general form, we can write

$$D = \lim_{R \to 0} [\, \log N \,/\, \log R \,] \qquad (1)$$

Parameters involved are

i) Delay τ as described.

ii) Embedding dimension.

iii) No of reference points.

The values of the above parameters are same for calculation of D of each sample. Since the number of data points are too high in each sample, we have used a real-time analysis program to find D which takes a chunk of 20000 points for each run; and there are nearly 15 runs for the entire file of any one sample. In Fig.1 we have shown the detailed results

of calculations. Discussion on D and the program used to calculate it have been discussed in our earlier works [13]. In this paper, we have 15 Ds 9one from each run) for each file and we present the minimum (Min_D) and maximum (Max_D) of them for each sample in Table1 and values of parameters used in the calculation are given in Table 2.

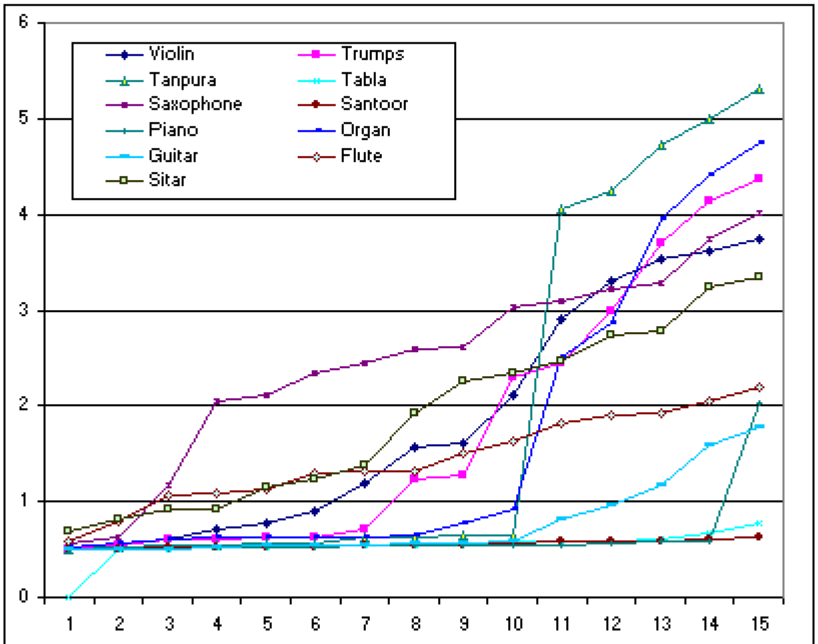

Fig. 1 : Details of D (vertical axis) calculated for each of the musical instruments. Horizontal axis shows run nmber.

Table1: Showing Max & Min D for instruments.

Instrument	Max_D	Min_D
Santoor	0.62	0.51
Tabla	0.77	0.50
Guiter	1.78	0.51
Piano	2.02	0.50
Flute	2.19	0.79
Sitar	3.35	0.81
Violin	3.74	0.53
Saxophone	4.02	0.62
Trumps	4.37	0.51
Organ	4.75	0.53
Tanpura	5.30	0.52

Table2: Showing values of parameters (with usual meaning as discussed in Sec 3.1) used in calculating D.

Embed. Dim	Delay	Ref. Points	Data points
8	4	512	20000

3.2 Spectral Analysis

One interesting feature that is to be noted. Spectrogram analysis show that even a highly trained voice produce several frequencies simultaneously due to the very nature of human voice production. For more details on this, one can see, for example, work done in [14]. Here for musical instruments also, same feature take palce- that is- more than one frequency at any instance of time as shown clearly in Fig 2.

As mentioned earlier, shifting from the time domain evolution of the recorded music to frequency domain gives us some opportunity to study the signals more closely. As 1/f nature of the music demands, at higher frequencies, amplitude falls to zero gradually. Also the entire musical phenomena is limited in the amplitude range, in log(db), of -30 to -70. As predicted theoretically, below -10, the signal turns to be white noise. These facts are nothing new, but for so many spectrums of the samples studied in this paper, as plotted in Fig. 3 confirm this again.

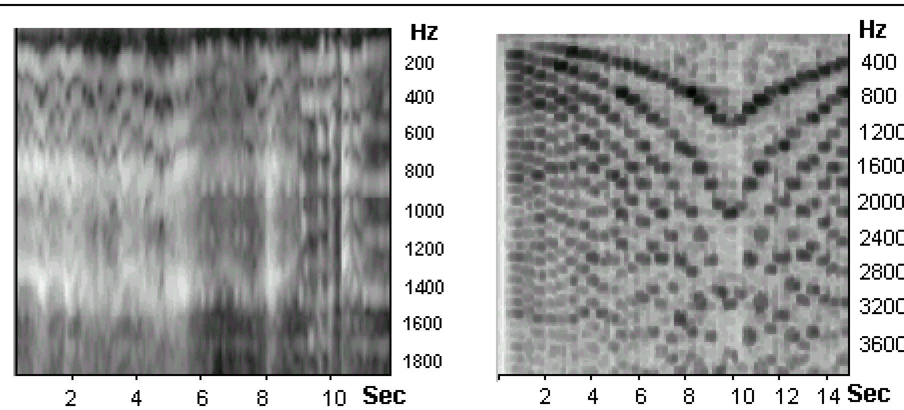

Fig 2: Neither human voice (left) nor musical instrument (right) can produce a single frequencty at a time. Spectrum Analyzer generated figure for a classical male singer and violin performance.

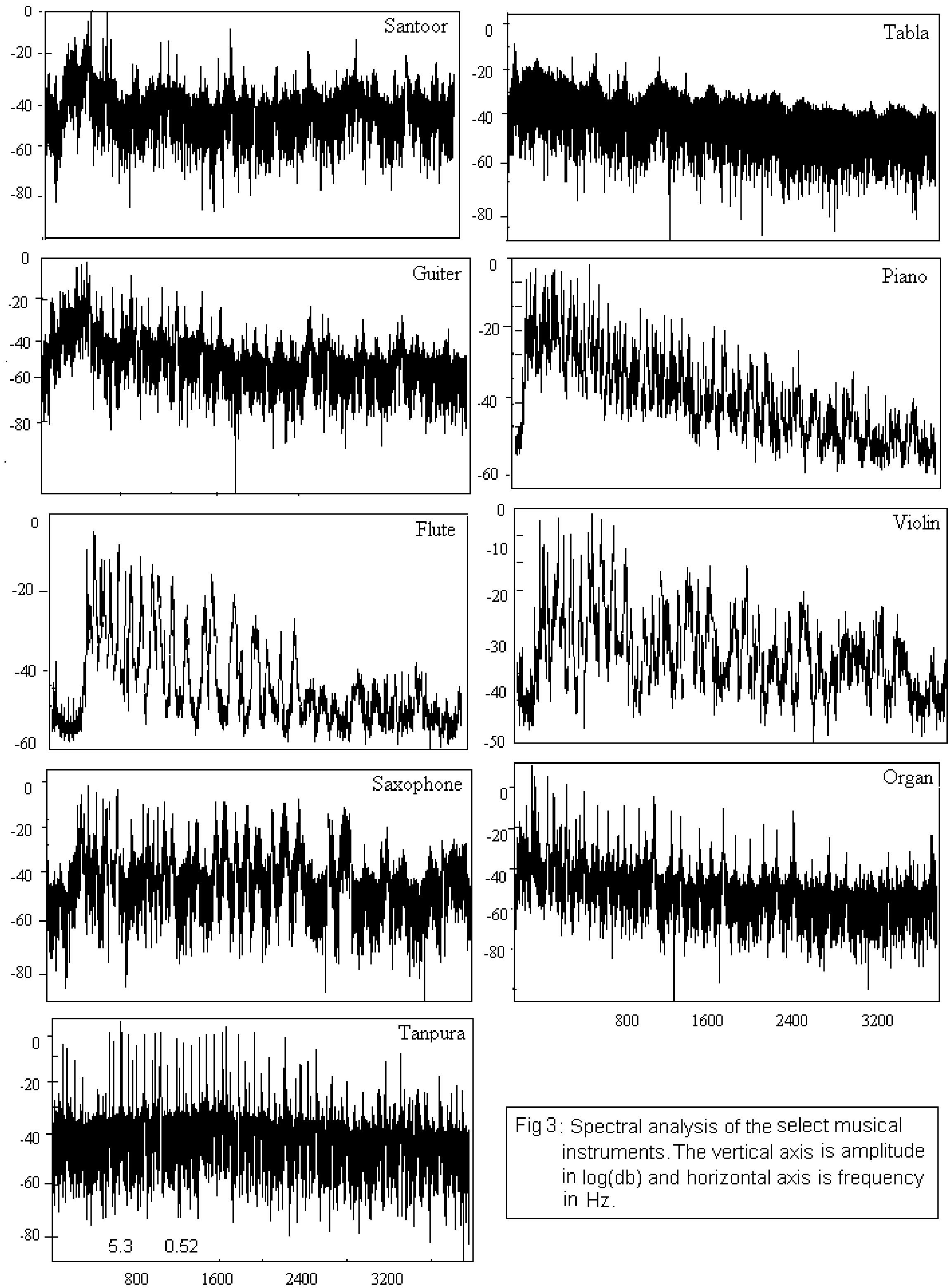

Fig 3: Spectral analysis of the select musical instruments. The vertical axis is amplitude in log(db) and horizontal axis is frequency in Hz.

4. Results:

A). The values of D are different for different types of instruments. But, interestingly, the Min_D calculated is never below 0.5 for any instrument.

B). For some instruments, the Max_D is quiet higher than others. In fact as the Fig. 2 shows, we find 3 groups of instruments for whom the Max_D is

i) around 1: santoor, sarod, tabla
ii) around 2: guitar, piano, flute, sitar
iii) around and above 4: violin, saxophone, tanpura

C). In general, the values of D cannot be correlated to the type of instrument for which it is measured. For example, different types of string instruments under study, like guitar, violin, tanpura have different Ds. The same is true for percussion and wind type instruments.

D). In our case, the value of (Max) D for violin (3.74) is higher than that for flute (2.19). This is quite expected as a flute generates relatively few harmonics, whereas a violin generates many. The amount of harmonic content depends heavily on the instrument and its construction, materials used etc. (\Imp basic notes Fourier in music.htm)

E). As discussed earlier that musical instruments does not repeat the fundamental tone only. Rather harmonics are present showing as peaks in the spectral analysis of the musical pieces.

F). D measures how much a curve is 'kinky'. That is, a curve, which has several peaks has higher D. If we compare the frequency spectrum of tanpura (Max_D = 5.3) and santoor (Max_D = 0.52), we find numerous peaks in the former causing a much higher D. The peaks represent the harmonics which make the sound more pleasant. So we can conclude that instruments with higher D has more harmonics which may make its sound more pleasant.

5. Discussion

For a more in-depth study in the line presented in the paper, more samples are to be taken and they should be longer in time length. This task demands huge computational power. Music is neither a purely physical nor a purely philosophical phenomenon. Though not easy to quantify, still quantification is aimed at gaining at a better understanding of music. We believe that brief idea sketched in this paper may a step towards that goal.

References:

[1] M. Stephanie & M. Saffle, (1994) “L-Systems, Melodies and Musical Structure.” Leonardo MusicJournal 4:31-38.

[2] R. F. Voss and J. Clarke, “ '1/f noise' in music and speech,” Nature, Vol. 258 (1975) 317-318.

[3] R. F. Voss R. F. and J. Clarke, “'1/f noise' in music: music from 1/f noise,” J. Acc. Soc. Am., Vol. 63 (1978) 258-263.

[4] M. Bigerelle and A. Iost, “Fractal dimension and classification of music,” Chaos, Solitions & Fractals, (2000) 2179-2192.

[5] J. Hemenway, “Fractal Dimensions in the Music of Mozart and Bach,” The Nonlinear J., Vol.2 (2000) 86-90.

[6] K. J. Hsu and A. J. Hsu, “Fractal geometry of music,” Proc. Natl. Acad. Sc., Vol. 87 (1990) 938-941.

[7] P. Herrera, X. Amatriain, E. Batlle & X. Serra (2000), "Towards instrument segmentation for music content description:a critical review of instrument classification techniques", Proc. Intern. Symposium on Music Information Retrieval.]

[8] B. Manaris, C. McCormick & T. Purewal, (2002) "Can Beautiful Music be Recognized by Computers? Nature, Music, and the Zipf–Mandelbrot Law," College of Charleston, Technical Report CoC/CS TR#2002-7-1]

[9] A. Das & Pritha Das, “Classification of Different Indian Songs Based on Fractal Analysis,” Complex-System, Accepted, [MS WR #262271].

[10] Freely downloadable vocal, instrumental classical Indian music clips are available at http://homepage.mac.com/patrickmoutal/macmoutal/rag.html

[11] Jan Beran (2004), "Which aspects of music can be described by quantitative models? Music - Chaos, Fractals, and Information," Chance, Vol. 17, No. 4.

[12] G. C. W. Sabin and D. Summers, “Chaos in a periodically forced predator-prey ecosystem model,” Math. Bioscience, Vol. 113 (1993) 91-113.

[13] A. Das, Pritha Das and A. B. Roy, “Nonlinear Data Analysis: A comparison between experimental [EEG] data and Theoretical [ANN] Data,” Complexity, Vol. 7, No 3 (2002) 30-40.

14] R. SenGupta, N. Dey and D. Nag, “Comparative study of fractal behaviour in quasi-random and quasi-periodic speech wave map,” Fractal, Vol. 9, No. 4 (2001) 403-414.

Appendix A

A short description of some Eastern musical instruments studied
Tanpura - long-necked ancient lute of India; four strings; provides the reference point for melodic improvisations by performers of the other instruments.
Sitar - Indian Classical stringed instrument (also has Persian links), modern type has 7 plucked strings and other sympathetic strings (not plucked); fretted with a gourd base; plectrum (misrab) can be used. There is a simple three string version called Setar in Iran.
Santoor - stringed instrument originating in Iran, belonging to the category of zithers; struck with two hammers.
Tabla - (or tabla-bayan) - NB Indian: an asymmetrical pair of small, tuned hand played drums (of the kettle-drum type) of north and central India, Pakistan and Bangladesh; the tabla drum is of wood, the Bayan of metal.

P. McClelland, Glossary of Folk Musical Instruments & Styles from Around the World, Hobgoblin Music at http://www.hobgoblin-usa.com/info/glossary.htm