A note on fixed points of generalized ice pile models

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Extended abstract

Ice Pile Models (IPM) were introduced in [1] to study the lattice of integers partitions. They are also a perused model for studying the behavior of systems governed by self-organised criticality (*SOC systems*). IPM can be seen as an extension of Sand Pile Models (SPM). SPM are a simple model based on a single local rule (*vertical rule*): a sand grain falls on its right if the difference its sandpile and the one on its right is bigger than a certain amount of grains. In the case of IPM, one more rule is added (*horizontal rule*): a grain slides to the right when there is a plateau.

The horizontal rule is not a local one. For this reason, in [4], the authors have introduced a variant for bounding the radius of the horizontal rule. They denote IPM(k), an IPM system in which a grain can slide to the right of at most k grains. This is the variant that we are going to discuss in the present paper.

Both SPM and IPM have been mathematically characterized(see [3] for a rather complete survey of known results). In particular, it has been proved that both systems have fixed point behavior and explicit formulas for the fixed points have been found. The issue is that these results are given only for a special initial configuration: all grains are concentrated in a single pile (column).

In [2], the authors exhibited a fast algorithm for computing fixed points of SPM under general initial conditions. They also claimed that it could be easily adapted to IPM. The present work aims at giving a formal proof to that claim.

References

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