COMPLEXITY MEASURES IN MANUFACTURING SYSTEMS

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Abstract

This study analyzes the most widewpread methodologies available in literature used to measure complexity. The research moves from a theoretical physic perspective, through the Complexity Theory, to a manufacturing system. On these subjects, two classification frameworks are proposed in order to categorize the most widespread measures. In particular, the second classification framework regards entropic measures widely used to measure complexity in manufacturing systems.

With reference to this second framework, two indexes were selected (static and dynamic complexity index) and a Business Dynamic model was developed. This model was used with empirical data collected in a job shop manufacturing system in order to test the usefulness and validity of the dynamic complex index.

The Business Dynamic model analyzed the trend of the index in function of different inputs in a selected work center. The results showed that the maximum value of the dynamic complexity index represents the so called "edge of chaos", where the amount of information needed to manage the system is maximum and where there is the trade off between flexibility and efficiency of the production system. In conclusion, the main result reached in this study regards the "edge of chaos" that is the target configuration for a company, in a particular system and under the same external conditions.

Key Words

Complexity Measures, Entropic Measures, Manufacturing Systems, Job-shop, Business Dynamics

1. Introduction

The origins of the studies of the Complexity Theory come from the researches about far from equilibrium thermodynamical phenomenon carried out by Nobel Prize Ilya Prigogine [1,2]. The following studies about complexity took very different directions and their development has been rushing and untidy because of their extreme multidisciplinary.

A system is a whole of linked parts which interact each other. Therefore, the complexity of a system refers to the number of connections or influences between the same parts of the system [3]. A “simple” system may assume a limited number of conditions, while a chaotic one may assume an enormous number of conditions because its parts are dispersed and they interact freely; in this way his behavior is absolutely not predictable. But a complex system is not a chaotic system. Particularly, a complex system [4] is made of a number of different parts, which possess specialized functions. The elements of the system are hierarchically organized and they are linked by many non-linear connections but the hierarchical structures guarantees to keep a kind of control. These non-linear connections make impossible an analytical approach for the description of every part of the system, while it is necessary a synthetic approach for the comprehension of the whole system. Therefore, a complex system places itself between systems whose behavior is simply predictable and the chaotic systems.

A particular kind of complex system is a complex adaptive system (CAS) [5]. This kind of system has another important characteristic: it changes, learns and evolves passing through “almost equilibrium” configurations. Complex adaptive systems are characterized by an emergent behavior of its elements, whose behavior stands between predictability and unpredictability.

Classical economic theory describes firms as entities whose target is optimizing resources utilization and maximizing earning [6]. Moreover, a company:

- Knows all available techniques, e.g. all possible combinations of input and output;
- Knows its own production function, e.g. it knows the maximum reachable level of output for every level and combination of input;
- Knows the costs of the factors, so it can define how to use at best productive factors which correspond with minimum costs for every desired output level;
- Knows market demand and so it decides the production level which corresponds with the output satisfying firm’s target.

In other words, a manufacturing system is a “device” which receives signals from the outside and, according to this signals, takes actions (for instance plans and starts the production activities). According to the classical paradigm, companies are simple systems in a simple environment and the relationships between their agents are simple and known. The instability is mainly due to the management ignorance or incompetence.

Complexity theory considers unsteadiness as a system’s characteristic which may have a random behavior even if no change happens outside: it is the structure of the system itself which generates the behavior represented by the science of retroactive control. Firms are clearly guided by retroactive control mechanisms.

A simple representation of manufacturing firms is no more possible because it doesn’t consider all decisions which have to be taken about the use of manufacturing factors: they have to be used in the best way, which must be also mutually compatible [6]; moreover, we can find the following characteristics:

- <u>Interdependence</u>: subsystems are linked each other, so the consequence of an action made on a unit depends of the actions made at the same time on the other units. There may be different kind of interdependencies, so their treatment may be more or less simple. We have a quite simple case when the relations between a unit and a group of other units are additive and separable (for example a linear linkage). In other cases interdependencies are more difficult and they are characterized by non linear relations, typical of a complex system.

- Numerousness of the states that units can assume (dimensionality of the system): the number of states that a unit may assume depends of technical or organizational discretionary power they have. If we consider together numerousness and interdependence, we understand that if we intervene on a variable controlled by a unit or a subsystem, we may find states very different from the starting one. This kind of difficult is one of the major problems to face in the management of complex systems.
- Uncertainty: in complex manufacturing systems, the outside conditions the system must face and the states the system may assume are never completely predictable.
- Irreversibility: it means that there is a cost attached to the changing of state, often associated with uncertainty.

The comprehension of the existence of these features allows us to consider organizations as complex systems in complex environments, which have to be managed with complex managerial methods.

2. Complexity Measures

Complex systems are characterized by such a number of different aspects which makes extremely difficult their study and many doubts arose about the possibility to create a unifying theory about complexity [7]. It is very hard define what a complex system exactly is, but it is even harder trying to develop a complexity measure which can consider all different aspects of complexity. Such a difficult is related to the multidisciplinary of such theory, which made the substantive “complex” assume very different meanings [8]. “Complexity” itself has been used in the wrong way by many authors, who wanted to bescribe properties which have nothing to do with complexity theory [9].

Different works have been examined to create an overview of the literature. The works examined were 105 and they were published in the period 1948-2005. These are the classes of the works:

- Papers and international reviews (34 papers)
- Collections of essays or book chapters (7 papers)
- Workshops and conference proceedings (42 papers)
- Papers on line (22 papers)

The analysis concentrated on two main aspects. In the first phase we studied the theoretical basis of complexity measurement, and we examined arguments as:

- Physics and Theoretical physics (25 papers)
- Information theory (5 papers)
- Classical science (8 papers)
- Scientific applications (8 papers)

In the second phase we studied the managerial applications of these studies, whose arguments are:

- Manufacturing (28 papers)
- Supply chain (13 papers)
- Design (3 papers)
- Corporate organization (1 paper)

Proposed Classification of Complexity Measures

Considering the researchers point of view, we found in the literature two kinds of divisions for complexity measures. If we consider the focus of the measure, we can distinguish:

- Deterministic complexity: the focus is on the random behaviour of the system. Such complexity measures are maximized for random strings.

– Statistical complexity: the focus is on the structure of the system.

If we consider the methodology used to develop the complexity measure, we can find two classes:

– Computation theory: it is needed a mechanism, usually an Universal Turing Machine (UTM) to calculate the measure;
– Information theory: they are connected with Shannon' entropy formula.

If we cross these variables, we obtain the matrix in figure 1. The terminology is the same used by the Santa Fe approach (see [9]) and [10]). These authors state that:

> "The quantities that have been proposed as general structural measures are often referred to as complexity measures. To reduce confusion it has become convenient to refer to them instead as statistical complexity measures. In so doing they are immediately distinguished from deterministic complexities, such as the Kolmogorov_Chaitin complexity, which requires the deterministic accounting of every bit – random or not - in an object."[9].

For example, in the first quadrant (high – left, Deterministic complexity – Computation theory) we find the "classical" complexity measure, e.g. Kolmogorov complexity, which has been developed to adapt Shannon's information theory to dynamical systems study. We define it using the word of [9].

> "The Kolmogorov-Chaitin complexity K(x) of an object x is the length, in bits, of the smallest program (in bits) that when run on a Universal Turing Machine outputs x and then halts."

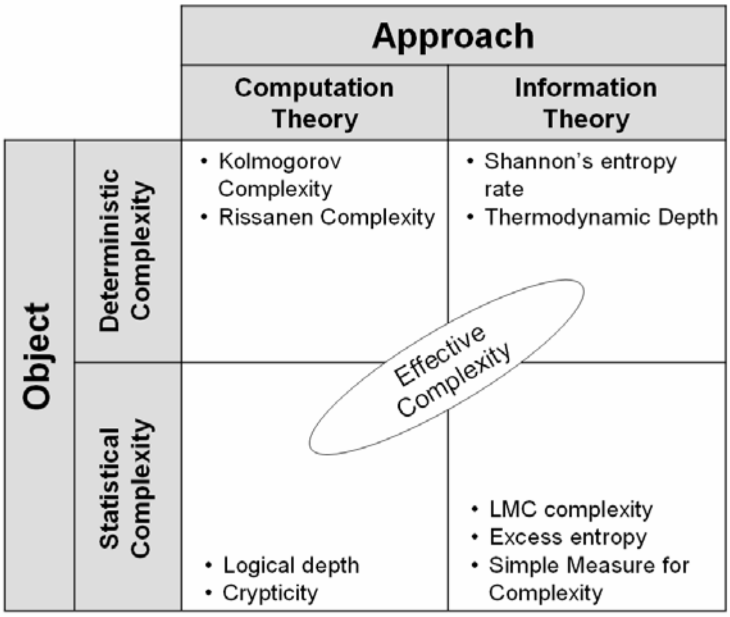

Figure 1: Proposed Classification of complexity measures

This kind of measure is maximized by random strings. During the years which followed the birth of this measure, it has been better appreciated that measuring the randomness of a system is not sufficient to understand all the aspects linked to systems' complexity [9]. Moreover, this quantity is usually not computable [11]. Therefore we can say that Kolmogorov complexity is an ancestor of complexity measures, although it is useless in many practical problems.

An example of deterministic complexity which makes use of information theory is metric entropy, which can be defined as a measure of the degree of long term unpredictability of a system.

Table 2 is a synthesis of the main literature references analyzed. It points out some characteristics of the measures analyzed together with the synonyms found in the literature.

Some deterministic complexity measures, which make use of computation theory, have been developed before the birth of Complexity Theory. These measures have been developed to characterize:

> "[…] deterministic sources of apparent randomness in the late […]. These efforts to describe the unpredictability of dynamical systems were largely successful." [9].

When the study of complex systems were still at the beginning, the main feature that captured researcher's attention was the apparently random behaviour of many natural systems. Therefore it has been immediate to use the existing "randomness" measures.

In the following years the study of complex systems made many steps further:

> "Since that time though it has become better appreciated that measuring the randomness and unpredictability of a system fails to adequately capture the correlational structure in its behavior" [9].

Therefore these kinds of measures, although historically important, are "incomplete" complexity measures. Similarly, the main limit of deterministic complexity measures which make use of information theory lays in the nature of such measures, which don't consider the structure of the system. Because of the uncomputability associated with UTM, the utility of statistical complexity measures which make use of the computation theory is limited to the possibility to get information about the system analyzing the way these measures converge [12].

In the last years, the main interests have been concentrated on statistical complexity measures which make use of information theory. These measures combine an entropic index, which describes the randomness of the system, with a corrective term. The aim is to comply with boundary conditions, where complexity measure must lower for perfectly ordered systems or random systems.

As [13] writes, the main limit of information theory is that it doesn't give information about the structure of the process, it is a probabilistic description of its behaviour. Moreover,

> "Information itself is never rigorously defined; it is only quantified." [13]

Therefore the quantity of information is identified, not the nature. However it's appropriate to emphasize that the entropic measures are used more often than other types of measures because:

> "There are many ad hoc methods for detecting structure but none are as widely applicable as entropy is for indicating randomness." [14]

Finally the "complexity" of the system (the texture of interactions between the constituent elements) generates a distribution of probability that describes the possible states that the system can assume. The entropy function elaborates such distributions of probability.

Table 2: Complexity measures and characteristics

Name	Authors	Period	Characteristics	Synonym
Shannon's entropy rate	Shannon	end '40 - beginning '50	– It is maximum for equiprobable states, e.g. for chaotic systems. – Focus on system's behavior.	– Metric entropy
Kolmogorov complexity	Kolmogorov	'50	– It is needed a UTM. – Incomputable. – Maximized for random strings.	– Algorithmic complexity; – Algorithmic information content (AIC); – Algorithmic information; – Algorithmic randomness; – Dynamical entropy; – Kolmogorov – Chaitin complexity (or entropy); – Kolmogorov – Sinai
Excess entropy	Packard, Crutchfield	1982	– The process complexity depends by the capability to foresee the behavior when the available information increase.	– Stored information; – Effective measure of complexity;
Logical depth	Bennet	1986	– Computation time measure of system's structure. – It measures the difficulty of making predictions from theory.	
Thermodynamic depth	Lloyd, Pagels	1988	– Complexity depends of the process which lead to the actual state. – Arbitrary choice of the states. – Although it has been introduced as a structural measure, it is maximum for a random behavior.	
Rissanen complexity	Rissanen	1989	– It studies the stohocastic behavior of the system. – Distinction between predictable and casual states. – Maximized for random strings.	– Stochastic complexity – MDL (Minimum Description Length); – Non-linear modeling
LMC complexity	López-Ruiz, Mancini, Calbet.	1995	– Complexity depends of entropy and disequilibrium. – Quite easy to calculate. – Formally not correct because of its non extensivity.	
Effective complexity	Gell-Mann	1995	– Its focus are both structure and behavior. – It depends of observation and observer conditions. – Complexity is created by element's connections.	
Simple measure for complexity	Shiner, Davison, Landsberg	1999	– It depends of a quantity called "disorder" (ratio between two entropies) – Formally not correct.	
Self – dissimilarity	Wolpert, Mcready	1997	– It requires the creation of a model from experimental observations. – It requires multiscale observations. – Particular case of effective complexity.	

Complexity Measures in Management

In the literature about management, two classifications have been found with reference to the measures of complexity of a manufacturing system (see table 3) (Gaio et al. [6] e Calinescu et al [15]). Gaio et al [6] identifies two principal group of measures for complexity:

- Fitness measures
- Entropic measures (static and dynamic complexity)

The fitness is the degree of adaptation of an organism in an environment (Darwin in 1868 speaks about "survival of the fittest"). In an evolutionary context the complexity generates advantages and possibilities [16] giving a parallelism between the concept of fitness and the concept of complexity.

> "Innovation allows an organism to adapt to a changing environment. Ordered behaviour and structure are necessary as a foundation for further evolution and in order to take advantage of regularity." [13]

In the following years the concept of fitness has been declined to the managerial and organizational studies about:

- Organizational development [17]
- Organizational structures evolution [18]
- Technologies selection [19]

Therefore the analysis remains mainly qualitative [6] with only one type of index developed: the index of fitness. Beside the difficulties to the computation of such an index, this methodology has been rejected because it was an indirect measure of complexity

The further distinction that the authors make about the entropic measures regards the static and dynamic complexity. The static complexity measures refer to the structure of the production operations:

> "[it] is a characteristic associable to the systems - and so also to the production processes - that refers to the structure of the facilities or to the structure of the plant and considers the degree of difficulty for their management and control. Such type of complexity becomes important when the possible design of a facility or plant is studied." [6]

Vice versa the dynamic complexity:

> " […] refers to the analysis of the systems along the time, in other words it studies the trend of the real states that the process assumes within the considered time. […] However from the point of view of the entropic measures we can consider […] the trend of the waiting queues (or the warehouses). In fact they absorb the variability of a system unit along the time" [6]

Calinescu et al [15] assert that:

> "So far, there have been a limited number of cases of using entropy for assessing, comparing or controlling manufacturing systems."

The authors distinguish four different variants of entropic measures:

- Deshmukh [20];
- Frizelle [21];
- Karp e Ronen [22];
- Yao [23].

Table 3: Most widespread classification framework in the literature

Author	Class of measure	Variant	Goal
Gaio, Gino, Zaninotto (2002)	Fitness		
	Entropic Measures (Information Theory)	Static Complexity	Study of the structure of the production operations
		Dynamic complexity	Study of the behavior of the production operations
Calinescu et al (2000)	Entropic Measures (Information Theory)	Deshmukh Approach (1993)	Structural Complexity Assessment
		Frizelle Approach (1995)	Structural and Dynamic Complexity Assessment
		Karp e Ronen Approach (1992)	Information generated by lot production
		Yao Approach (1985)	Routing Flexibility Assessment

The entropic measures have been already analyzed within the theoretical physics (see previous section). According to the provided analyses, entropic measures appear as the most used methods to measure complexity. Figure 2 shows the main entropic measures founded in the scientific literature. In particular, we named them as follow:

- School of Cambridge - Oxford: the starting point of the co-ordinated research project of the two universities is based on a first work of Frizelle [21]. Two types of measures, both entropic, are used in order to define:
    - Static complexity (function of the structure of the production operations);
    - Dynamic complexity (function of the behaviour of the production operations)

These measures define the uncertainty level, or quantity of information necessary, in a system description.

- Deshmukh [20]: this approach aims to analyze the static complexity in a production system using an entropic measure.
- Karp e R. Ronen [22]: the objective of the study is to demonstrate by the use of an entropic equation, that the smaller production lots requires less efforts in the management of the production system because the system requires a smaller quantity of information.
- D. D. Yao [23]: this study examines the “dynamic parts routing” inside the FMS (Flexible Manufacturing Systems). In order to measure the flexibility, the author develops an entropic-based measure combining the characteristics of the equipments and the characteristics of the systems which contribute in the flexibility of the routing.
- V. Kumar [24]: this research attempt to develop some measures of flexibility; an entropic measure of flexibility is developed.
- G. Allon, D. P. Kroese, T. Raviv, R. Y. Rubinstein [25]: the authors propose an optimization algorithm based on an entropic formula in order to solve the problem of buffer allocation inside a production system.
- Nam P. Suh [26]: this approach aims to measure the complexity in the design. The study is based on a technique of the Axiomatic Design in order to succeed to calculate the probabilities that a functionality of the product is defined in a satisfactory way from the

design parameters; in particular, this entropic measure has been used in order to define a complexity index.

- Janow [27]: the author aims to extend the use of some elements of the information theory (like the maximum transmission capacity of a channel) together with the Shannon entropic measure. The goal is the explanation of the behaviour of the large organizations, in which the nodes (persons) through which the information pass are numerous.
- R. Johnston [28]: the author uses an entropic formula in order to demonstrate that the impossibility to satisfy the market by using unitary production lots generates the minimum level of costs that are not dismissible.

Topic	Authors: Oxford - Cambridge 1995	A. Deshmukh 1993	A. Karp e R. Ronen 1992	D. D. Yao 1985	V. Kumar 1985	Allon, Kroese, Raviv, Rubinstein 2004	R. Janow 2004	Nam P. Suh 2004	R. Johnston 1998
Supply chain	13								
Production (Product-Process)	16	2							
Production (Lot Production)			2						
Flexible Manufacturing Systems				1					
Flexibility in production					2				
Buffer localization						1			
Production Organization	1								
Organizational Development							1		
New product Development	1							1	
Market									1

Figure 2: Classification of Entropic Measurements

The proposed classification of entropic measures in the production management

In order to give homogeneity and simplicity, the proposed classification of the entropic measures considers only those measures that refer to a productive context (figure 3). The selected variables for the classification are tasks of the studies and the objects of the mathematical formulation.

The fundamental objective at the base of the study influences the type of developed index. As an example, although the existing classifications indicate the measure of Yao [23] like one measure of complexity, we consider that it is a measure of flexibility used in order to take decisions in real time in a FMS when the lots advance.

The considered tasks are:

- Complexity, defined as the quantity of information necessary to describe the system. The research of Frizelle [21] and Deshmukh [20] explicitly regard the study of the complexity of production system.
- Lot sizing: the research focus is to demonstrate that the smaller production lots requires less efforts in the management of the production system because the system requires a smaller quantity of information.
- Flexibility: two main entropic-based measures of complexity regarding the flexibility has been highlighted:
    - Kumar [29] simply aims to develop a measure able to quantify the flexibility of a production system.

- o Yao [23] inserts his own measure of flexibility inside a methodology that can be used for the production plan in order to take decisions in real time.

- Buffer allocation: the authors use an entropic formula inside an algorithm of optimization for the buffer allocation.

		Objects			
		Resources State	Number of Products Production Time	Structure of Production Operations	Demanded Performance Equipment State
Tasks	Complexity (*)	FRIZELLE, 1995 DESMUKH, 1993			
	Lot Sizing		KARP & RONEN, 1992		
	Flexibility			KUMAR, 1986 YAO, 1985	
	Buffer allocation				ALLON et al., 2004

(*) Complexity: amount of information needed to describe a system

Figure 3: Classification of Entropic Measurements in Production Management

The parameters of the mathematical formulation represent the nature of the variables which appear in the proposed formulations. In particular they can be distinguished:

- State of resources:
    - o Frizelle [30] recognizes that the most important point of view for a production system is the dynamic complexity and considers the system composed by the machine and its queue as a resource. The studied state is the queue level.
    - o Deshmukh [20] defines a measure of static complexity that is a function of the requirements components' processes to produce and the equipments characteristics.
- Karp and Ronen [22] define an entropic measure that is function of parameters like the number of stations of the production line, the number of codes that must be produced for a certain product, the number of codes for lot, the number of lots and the relationship between the gross time of production (calculated as working time + queue time in buffer of the finished products) and the time (only process lead time) in the case the entire quantity is produced in a single lot.
- Structure of operations in production:
    - o Kumar [29] considers the probability that a production is realized in a determined working station.
    - o Yao [23] considers the probability that a prodution is realized in one determined working station considering also the probability that the machines used for successive operations could be out of order.
- G. Allon et al. [25] elaborate an entropic formula function of a sequence of accidental sampling (on the states of *n* machines) and the probability distribution of an accidental sampling (on the states of *n* the machines) function of a performance parameter.

3. The job-shop production system as a complex system

The fragmentation of the market and a wider push from part of the companies to an high customization of the product force firms to propose a wide and more differentiated range of product; in particular, the phenomenon that has pushed the companies to make the manufacturing planning and control system more complex is the so called *mass customization* [30]. In fact, the mass customization foresees an increase of the product variety. This variety growth aims to satisfy the requirements of the greatest number of customers but it corresponds to an increase of the number of codes and processes that a firm has to manage. The companies try to determine the right level of variety that they should have to offer to the market in order to optimize the profits [31].

In this context, many products do not have a sufficient market demand in order to justify the creation of a production line that is instead a production system more correct for products barely differentiated and with high volumes. The answer to different needs, corresponding to product with low market demand and which requires variations in short times, is given by the job-shop production system.

The complexity that comes from the variety is caused by the uncertainty generated by the interaction of the variety along the time making unforeseeable the behaviour of the production systems [30]. The present work focuses on the process of management and manufacturing planning in the job-shop production system.

In this type of plant the lay-out is defined by areas (functional units) in which homogenous machines are grouped in accordance with the function and operations carried out. In this type of process, a “shop” of a manufacturing firm can contain for example lathes, presses or machines for tests (see figure 4). Parts of a product assigned to the same work centre can demand the same type of production with different setups. A job-shop production system is designed above all for a particular category of products: the assembly components; very often these are objects that are assigned to the successive phase of assembly in order to obtain the finished product; the range of different component that can be obtained is very large [32].

The material flows generated by manufacturing operations is very articulated; therefore, it’s necessary an elevated capability to process the informative flow, in order to coordinate the production flows and control the work in progress. The informative flow constitutes a crucial point of the production system.

Manufacturing control and information flow management are strongly linked to the raw materials management, stocks management and operations scheduling. In fact, the controller needs the information about the manufacturing plan, the stocks levels, the labour capability, the advancing state of the job order and to their completion times, in order to allow the managers to formulate the production scheduling and to face problems which can rise from its application [33].

The manager needs information regarding the manufacturing requirements of every job: the delivery date, its position inside the production system, the working times, the queue time, the set-up times, the raw materials necessary to produce the demanded product, etc. All these information together with to the priority rules, are used to estimate the times of arrival to the successive work centre and to foresee possible material requirements in the warehouse [6]

The empirical study realized concerns a manufacturing firm with job-shop production system without orders scheduling. With reference to the formulation proposed by Frizelle [30] the study has been articulated in two phases:

- Estimation and analysis of the indexes of static complexity in a production unit (shop);
- Estimation and analysis of the indexes of dynamic complexity in a work centre and simulation of alternative scenarios through a Business Dynamics approach.

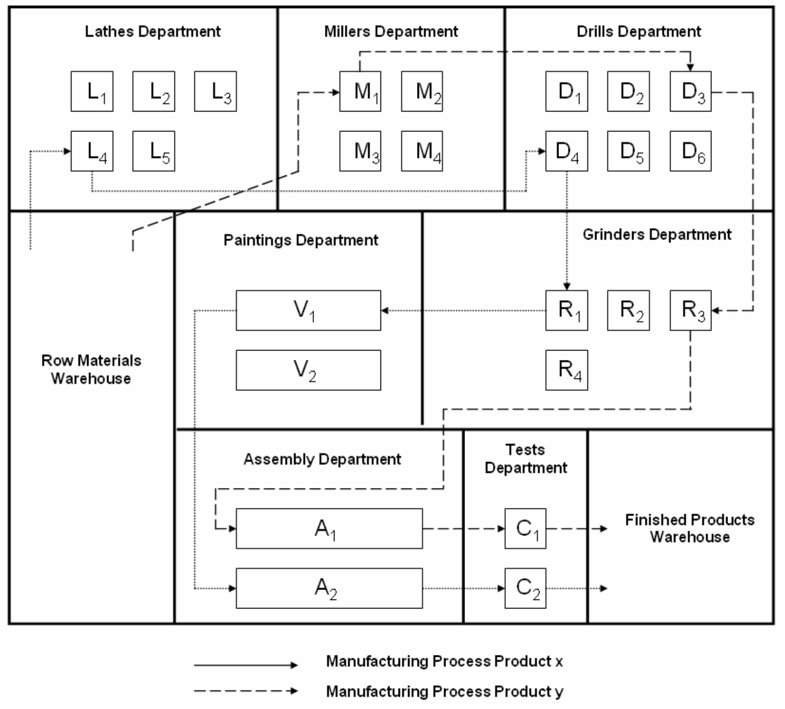

Figure 4: Production flows in a Job-Shop manufacturing system

The originality of the proposed study is in the use of complexity indexes through the System Dynamics approach. The analysis has been conducted using the data collected during the year 2004 integrated with random repeated observations. The difference between the approach suggested by Frizelle [30] and Calinescu et al [15] and this research refers to a production system in which the scheduling plan is not carried out. Therefore, it's necessary to carry out a series of adjustments to the methodology elaborated by Calinescu [15].

<u>Static Complexity index</u>

The static complexity starts from the competition between products and resources; it is an index that represents the potentiality inside the structure to cause operative critical states. The Static Complexity index [30] is associated to the system variety linked to the planned state. The mathematical formulation is the following:

$$H_{static}(S) = -\sum_{j=1}^{M}\sum_{i=1}^{N} p_{ij} \cdot \log_2 p_{ij}$$

where M is the number of resources (that is the number of machinery, equipments), N represents the number of possible states the resource j can be found and p_{ij} is the probability that the resource j is found in the state i [15]. The authors define "planned state" the association between the product i and the resource j which will work according to the scheduled plan.

In the case study carried out the company doesn't schedule, so the indexes has been calculated using the consumptive data related to the working times of the products on the different machines present in the company database about year 2004. In fact, static index of complexity can interpreted as a representation on the resources by different products. In accordance with the assertion by Frizelle [30] and Calinescu et al [15] these resources are those that potentially could be more critical to manage.

For every resource the time committed by every product has been calculated and then we determined p_{ij} as the ratio between the time within the product has committed the resource j divided to the total time in which the resource has worked during the year.

Static complexity index has allowed to identify the most saturated work centre, characterized by the biggest indexes. In reference with this particular application, the static complexity index hasn't provided any additional information, compared with a *traditional* index.

Dynamic complexity index

Dynamic complexity studies manufacturing operations' behaviour; it represents the quantity of information necessary to describe the state of the system when it diverges from planned states. It can be calculated through the study of queuing behaviour [15], using the formula:

$$H_D = -P \log P - (1-P)\log(1-P) - (1-P)\sum_{i=1}^{M} \sum_{j \in NS_i} p_{ij} \log p_{ij}$$

Where P is the probability of the system to be in a planned state (as described in system's schedule), while p_{ij} is the probability of the resource j to be in an "out of control state" i.

Literature's case studies ([34], [30]), always considered companies which used schedule software; in this way, it was always possible to know if planned state had been observed. Resource's state depends of the deviation between actual and scheduled state. In this way, work centre queues were observed through an indirect way, because it is supposed that a machine doesn't work what has been planned because of the unplanned behaviour of system's queues. In summary, the required data for index calculation are collected with precision, and researcher's subjectivity affects only states definition [35]. It is suggested [35] to group states in ranges depending of the "seriousness" of the gap between actual and scheduled state, where "seriousness" is the distance from the planned state of the resource. In this way it is possible to determine the "in control" and "out of control" states and calculate the indexes, which can be compared each other [35].

In this case study the production is not scheduled, so it is hard to determine in which way a resource is in a control or out of control state. This is why it is impossible to determine only one group of states which is the same for all resources and it is impossible to give only one definition of "in control". So, it has been necessary to adapt the approach we described before to study dynamic complexity in this case study.

The manufacturing system considered in this case study is a job-shop, so there is the typical problem of the trade-off between a saturation level which allows utilizing machines at best and a queue which doesn't extend lead time too much. The queue is the total quantity of work, expressed as standard hours, which lays in every moment by the work centre, waiting for being manufactured. The aim of the queue is to decouple work centres which have different cadence or lots with different routes (fig. 4). In job-shop manufacturing systems queue times plus moving times usually reach even 90% of system's manufacturing lead time. The research for the right trade-off between queue time and saturation aims to reduce at most queue without reducing productivity. Queues reduction implies shorter lead times and so an higher flexibility to react very quickly to market's demand.

The adaptation of the quoted approach wants to describe, for each resource, the behaviour:

- Saturation: the saturation curve will raises as the queue raises, till it will reach an asymptote which corresponds with the higher saturation level the resource may reach.
- Number of parts in queue: the number of parts is directly proportional with the queue time, if we consider a multiplicative constant which corresponds to the parts' manufacturing time.

The two curves will depend of the mean number of parts' arrival by the machine, called the system's input or simply input. The aim of the model is to observe the course of the two curves and the dynamic complexity index as the input varies. The advantage coming from simulation lays in the possibility to study conditions which will never be reached in reality.

The model

Now it will be described the Business Dynamics simulation model developed (Powersim® simulation software), which aims to reproduce the functioning of work centre. The study is based on the same criteria for each work centre, so it will be sufficient to describe the process and the results for only one work centre.

The model simulates the mean daily number of parts arrivals by the examined work centre and its manufacturing cadence. The fundamental parameters of the simulation have been calculated from firm's database; year 2004 data have been considered, and they are:

- Input level: the number of incoming parts varies within a fixed range, which has been calculated from historical data.
- Starting queue: its value is equal to the queue mean value which has been learnt through repeated random observations made for each work centre.
- Cadence: the mean of each work centre has been calculated through the 2004 data of the firm's database. The mean cadence varies in a fixed range which depends of unpredicted situations as machine breakdowns or failures. Moreover, cadences depend of the manufactured part and it results from the different manufacturing times.

Figure 5 schematically shows what we explained. The model aims to reproduce the queuing behaviour of the parts in work centre's buffer and it considers work centre's different operating conditions. Two models with different priority rules have been developed:

- FIFO rule (First In, First Out): the first part arriving will be manufactured first;
- Lots' joining rule: parts may be manufactured only when queues reach a dimension equal to manufacturing lot of the machine.

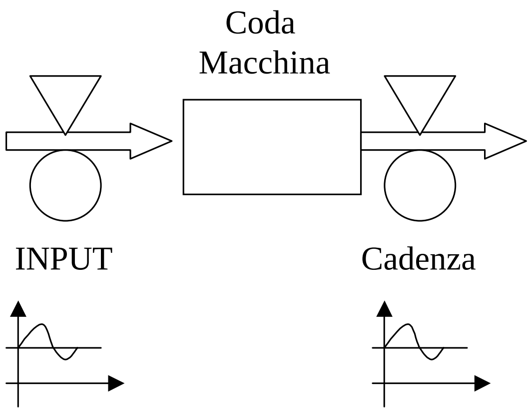

Figure 5: Simplified work centre model

As we told before, unlike literature's case studies, our data were referred to a firm which doesn't schedule. Therefore it has been necessary to adapt the state definition procedure described by [35].

A sequence of simulations was performed to obtain a reference queue level. In this way the curves of queue and saturation had been obtained in function of system's input. It allowed us to the queue level which corresponds to an optimal, or reference, functioning of every work centre.

The work centre queue level found has been utilized to define the "in control" state of the work centre. So, the state definition can be expressed in this way:

- In Control state: it is defined as an neighborhood of the reference queue level which had been calculated before;
- Not In Control states: we determined four out of control states with growing seriousness. They correspond to superior or inferior queue ranges respect to the reference queue state. For example, the out of control state 1 corresponds to a state which is few different from the reference state, corresponding to queue levels few higher or few lower than the optimal ones.

After the state definition, the complexity index has been calculated. For this purpose, a sequence of one year lasting simulations has been conducted for growing input levels. Simulations had been repeated for every input level in order to lower the dependence upon random variations of input and manufacturing cadence. The following values have been calculated for every simulation day:

- Mean queue: it is calculated on the entire simulation year;
- Saturation: it is calculated from the ratio between the parts manufactured during the simulation day (mean manufacturing cadence of the machine) and the maximum number of parts the machine can manufacture ("maximum" manufacturing cadence of the machine).

Probabilities P (probability of the resource to be in control, e.g. the queue is in the settled range) and p_i (i = 1,…,4 out of control states) have been obtained calculating the number of parts in the queue for every simulation day. So, it have been possible to count the number of days when the resource was in a determined state and then it has been obtained the probability of every state divided by the number of the simulation's days. The results for the FIFO rule are represented in Fig. 6 a and b.

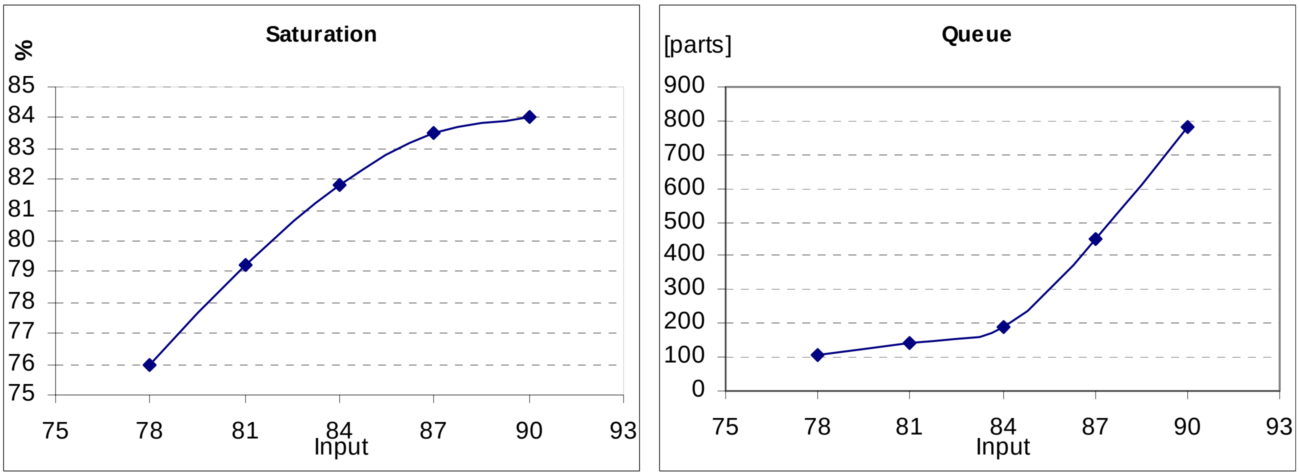

Figure 6a: Saturation and Queue Time in function of the input level

It is possible to see that complexity index curve is not increasing monotonic function. So, the index may have the same value for different input levels.

The dynamic complexity index curve has a maximum value for an input value equal to I*. If we use a managerial perspective, this value corresponds to the input level which is the most difficult to manage because it requests the higher quantity of information. At the same time, it is the most interesting if we consider the profitability of the work centre. In fact, in this situation the firm pursues the aim of reducing lead time, although maintaining a good saturation level. Consequently, the system is maximally complex because it has to satisfy opposite aims. If we consider the dynamical complexity index, it is possible to individuate three areas of the curve:

- Input levels inferior to I*: The resource has a low queue level, so it has a low complexity index because the out of control states it assumes vary in a strict group. On the other hand, the resource has low saturation, so it is possible to suppose a not optimal utilization of the machine. A firm which places itself in this zone has as main target the lead time reduction.
- Input levels superior to I*: The resource has a high queue level, so it has a low complexity index because it is out of control for the most of the time. Saturation is high and the machine has always available a parts buffer to be manufactured, minimizing inactivity time. A firm which places itself in this zone has as main target the saturation increase.
- Input levels close to I* (Edge of chaos): The firm looks for a queue level which allows it to satisfy opposite targets of lead time reduction and inactivity time decrease. A firm which

places itself in this zone has a double target (e.g. optimizing manufacturing lead time and machines saturation).

So the highest complexity index area appears as the most difficult zone to manage (managerially complex), but also as the most fruitful.

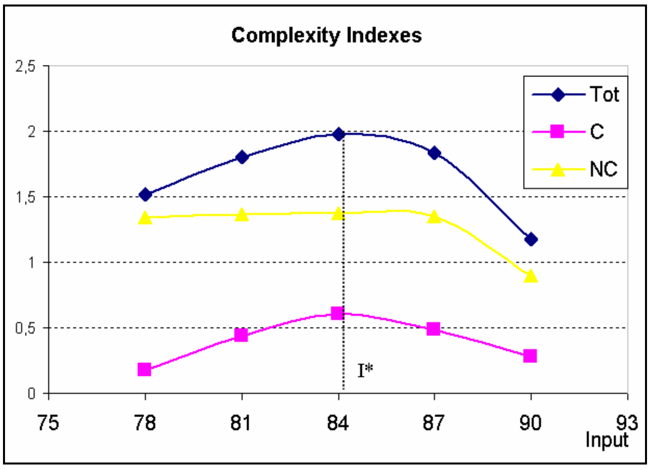

Figure 6b: Dynamic complex index in function of the input level

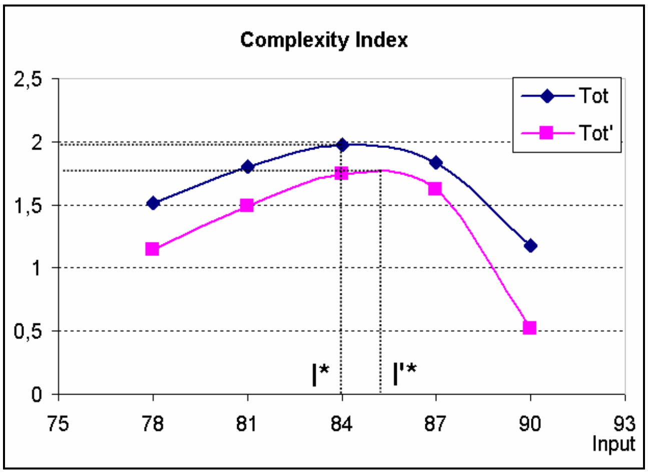

Figure 7: Dynamic complex index in function of the input level with two levels of input range (+/-25 e +/-15)

<u>Creating an alternative scenario</u>

The alternative scenario represents a firm which exercises a higher control on its manufacturing flows. As a consequence of a higher control, with the same manufacturing cadence, it was supposed that the input level would have a lower daily variation.

As in the first study, we made groups of 12 simulations for every growing input level. In this way it was possible to determine the new curve of the complexity index.

If we consider simulation results (fig. 7), we can see that the complexity index still have a maximum value for an input level equal to I'*. The maximum value of this scenario is anyway inferior to the maximum value obtained in the first study (higher input variation). Consequently, it is possible to agree with the statement that dynamic complexity can be managed with an higher control on manufacturing flows [30].

4. Conclusions

The literature review has drawn the attention to different methodologies available to measure complexity. In a theoretical perspective we classified the measures according to two dimensions: the object of the measure (deterministic or statistical complexity) and the approach (Computation or Information Theory). With reference to a manufacturing setting the framework proposed characterize different entropic measures according with two variables: tasks (complexity, lot sizing, flexibility or buffer allocation) and objects (resources state, number of products, production time, structure of production operations, demanded performance or equipment state).

Two indexes have been selected from this framework (static and dynamic complexity index) due to their consistency to a manufacturing setting. These indexes have been adapted to a job shop plant without schedule.

A Business Dynamic approach allowed to analyze the trend of the dynamic complexity index as regards to different operating conditions. The dynamic index was studied not only according to the empirical data, but also assuming supposed scenarios with different levels and range of inputs. Furthermore, the dynamic index was compared to “traditional” indexes as the queue time and the capacity saturation percentage.

In particular, the level of input for the peak of the dynamic index has interpreted according to the theoretical knowledge highlighted in the literature review after we shared the results with the practitioners in the company.

The comparison between the curve obtained and the saturation and queue indexes corroborated that the peak of the dynamic index corresponds with the trade off between flexibility and efficiency of the work centre studied. Moreover, this peak coincides to the edge of chaos, characterized by the maximum managerial complexity. Metaphorical speaking, the behavior of the dynamic complexity index may be described as following: if we move to the left section of the diagram (fig. 060b) (Input < I*) the capacity is unsaturated and the work center “dies of thirst”; On the contrary, if we move to the right section of the diagram, the capacity is saturated, the queues increase and the work center drowns.

The main limit of this approach regards the impossibility to compare directly complexity indexes of different work center. As a matter of fact, if the company schedules, we can imagine that the planned state is the optimal one. In the case studied, the “in control” and “not in control” were different for every work center.

In other words, the computed indexes are stand alone for each work center. For instance, if the complexity index is 1.5 for a particular system (work center, equipment, etc.) rather than 2.0, it doesn’t make nay sense.

It would be deliverable to identify a normalized complexity measure in order to compare different systems or work centers. The normalization could spring from the maximum level of complexity sustainable. According to Frizelle [30], this level of complexity coincides to the maximum complexity manageable for the organization

In this reference, if the complexity index is 1.5 for a particular system (work center, equipment, etc.) and the maximum sustainable complexity index is 2.0, it means that the organization needs further information in order to reach the optimal configuration.

References

[1] Prigogine I., From Being to Becoming: time and complexity in the physical sciences, Freeman, San Francisco, 1980

[2] Prigogine e Stengers I., La nuova alleanza. Metamorfosi della scienza, Torino, Giulio Einaudi Editore, 1981

[3] Celestino R., *Il manager in tempo di crisi*, in Ideediade, www.ipq.it/ideediade/n_25/management.htm, 2002.

[4] De Rosnay J. , *Il macroscopio. Verso una visione globale*, Bari, Dedalo, 1977.

[5] Holland J., *Sistemi adattativi complessi ed emergenza spontanea,* in Quadrio Curzio A. and Fortis M., *Complessità e distretti industriali. Dinamiche, modelli, casi reali*, Bologna, Il Mulino, 2002.

[6] Gaio L., Gino F., Zaninotto E., *I sistemi di produzione*, Edizioni Carocci, Roma, 2002.

[7] Horgan J., *From Complexity to Perplexity*, Scientific American, Vol 6, pp 74-79, 1995.

[8] Dioguardi G., *Al di là del disordine. Discorso sulla complessità e sulla impresa*, Napoli, CUEN, 2000.

[9] Feldman D. P., Crutchfield J. P., *Measures of Statistical Complexity: Why?* , Phys.Lett.A, Vol 238, pp 244-252, 1998.

[10] Gell-Mann M., *What is complexity?* , Complexity, Vol 1, No 1, pp 16-19, 1995.

[11] Shalizi C., *Complexity Measures*, available on http://cscs.umich.edu/~crshalizi/notebooks/complexity-measures.html, 2003.

[12] Crutchfield J. P., Shalizi C., *Thermodynamic Depth of Causal States: When Paddling around in Occam's Pool Shallowness Is a Virtue*, Santa Fe Insitute Working Paper 98-06-047, 1998.

[13] Crutchfield J. P., *Complexity: Order contra Chaos*, in "Handbook of Metaphysics and Ontology", Philosophia Verlag, Munich, 1989.

[14] Gell-Mann M., Crutchfield J. P., *Computation in Physical and Biological Systems Measures of Complexity*, available on http://www.santafe.edu/research/measuringComplexity.php, 2004.

[15] Calinescu A., Efstathiou J., Sivadasan S., Schirn J., Huaccho Huatuco L., *Complexity in Manufacturing: An Information Theoretic Approach*, Proceedings of the International Conference on Complex Systems and Complexity in Manufacturing, Warwick, pp 30-44, 2000.

[16] Schuster P., *How does complexity arise in evolution?* , Complexity, Vol 2, pp 22-30, 1996.

[17] Teece, D.J., Pisano, G. and Shuen, A., *Dynamic capabilities and strategic management*, Strategic Management Journal, Vol. 18 No. 7, pp. 509-33, 1997

[18] Westhoff F., Yarbrough B., Yarbrough R., *Complexity, Organisation and Stuart Kauffman 's "The Origins of Order"*, Journal of Economic Behaviour and Organisation, Vol 29, No 1, pp 1-25, 1996.

[19] Kauffman S., *Escaping the Red Queen effect*, The McKinsey Quarterly, Vol 1, pp 118-129, 1995.

[20] Deshmukh A., Talavage J., Barash M., *Complexity in manufacturing systems, Part 1: Analysis of static complexity*, IIE Transactions, Vol 30, pp 645-655, 1998.

[21] Frizelle G., Woodcock E., *Measuring complexity as an aid to developing operational strategy*, International Journal of Operations and Production Management, Vol 15, No 5, pp 26-39, 1995.

[22] Karp A., Ronen B., *Improving shop floor control: an entropy model approach*; International Journal of Production Research, Vol 30, No 4, pp 923-938, 1992 .

[23] Yao D. D., Pei F. F., *Flexible Parts Routing in Manufacturing Systems*; IEEE Transactions, Vol 22, No 1, pp 48-55, 1990.

[24] Kumar V. , *Entropic measures of manufacturing flexibility*, International Journal of Operational Research, Vol 24, No 4, pp 957-966, 1987.

[25] Allon G., Kroese D. P., Raviv T., Rubinstein R. Y. , *Application of the cross-entropy method to the Buffer Allocation Problem in a Simulation-Based Enviroment*, in print for Annals of operations research, 2004.

[26] Nam P. Suh, *Complexity: Theory and Applications*, Oxford University Press, 2004.

[27] Janow R., *Shannon Entropy and Productivity: Why Big Organizations Can Seem Stupid*, The Analytic Solutions Group, LLC, 2004.

[28] Johnston R., *From Efficiency to Flexibility: Entropic Measures of Market Complexity and Production Flexibility*, Complexity International (electronic journal), Vol 3, 1996.

[29] Karp A., Ronen B., *An information entropy approach to the small-lot concept*, IEEE Transactions on engineering management, Vol 41, No1, pp 88-92, 1994.

[30] Frizelle G., *The Management of Complexity in Manufacturing*, Business Intelligence, Londra, 1998.

[31] Martin M. V., Ishii K., *Design for Variety: Development of Complexity Indeces and Design Charts*, Proceedings of ASME Design Engineering Technical Conferences, Sacramento, 1997.

[32] Da Villa F., *La logistica dei sistemi manifatturieri*, Etas, Milano, 2000.

[33] Schmenner R. W., *Produzione. Scelte strategiche e gestione operativa*, Edizione Sole 24 ore, Milano 1987.

[34] Sivadasan S., Efstathiou J., Frizelle G., Shirazi R., Calinescu A., *A Methodology for Measuring Complexity in the Supply Chain*, International Journal of Operations and Production Management, 2000.

[35] Sivadasan S., Efstathiou J., Calinescu A., Huaccho Huatuco L., *A discussion of the issues of state definition in the entropy-based measure of operational complexity across supplier-customer systems*, Proceedings of The 5th World Multi-Conference on Systemics, Cybernetics and Informatics, Orlando, 2001.