# Fame as an Effect of the Memory Size [*]

Haluk Bingol

Department of Computer Engineering
Bogazici University, Istanbul 34342 Turkey
Tel: +90-212-359-7121
Fax: +90-212-287-2461
haluk.bingol@boun.edu.tr

**Abstract.** This paper investigates the effect of the memory size to fame. A population of individuals of the same memory size is considered. A simple recommendation model is defined based on the memory of the individuals, the social network and the population memory. Recommendation process changes the content of the memory. As the ratio of memory size to population size decreases, a self-organized pattern emerged in who-knows-who graph. Majority of the people become unknown yet a few become very famous. This could be a model for fame and memory.

**Key words.** Fame, popularity models, reference models, knownness, memory, social networks, self-organizing systems, network growth models.

## 1. Introduction

Suppose we need a dentist or need information about an item that is either a generic or a particular one. What do you do?

Our first approach is to check our memory whether we have any. If we do not know, then our second attempt would be to ask our friends for a recommendation. Then we would go to cataloged data such as yellow pages, search engines. A couple of concepts are involved in these processes including our memory, our social network, the cumulative memory of the social network, a recommendation process.

Suppose we are looking for *S*. The more *S* is known, the more it can be reached, since the probability of finding a person that knows it within our social network increases. Conversely, the less *S* is known, the more difficult to be reached. In the limit case, if nobody knows it, then there is no way to reach it. This discussion calls for fame which is the central concept of this paper.

[*] This work was partially supported by Bogazici University Research Projects under the grant number 05A105.

## 1.1. Fame

Wikipedia.org defines fame as **"**the condition of being known to the general public". Popularity, which is particularly difficult concept to measure, is investigated in various contexts. One pragmatic measure is the search results in the web. A Google search returns a number of web pages that refer to an item. The assumption is that the more pages return, the more popular the item is. This measure of popularity has an implicit problem since an item can be listed because of some other reason than what we are looking for. For example, a singer can be listed because of being a movie actor as well as being a singer.

The fame of WWI fighter-pilots obtained using this approach [1]. Consider the distribution of fame *p(f)* that is the probability of having fame *f*. It is found that *p(f)* decreases with a power-law of *f*, namely

$$p(f) \sim f^{-\gamma}\,; \quad \text{where } \gamma \approx 1.9.$$

Let *a* be the achievement of the person which is defined as the number of opponent aircrafts destroyed. It is found that *p(f)* increases exponentially with achievement *a*,

$$p(f) \sim e^{-\beta a}\;; \text{ where } \beta \approx 0.074.$$

In a similar study, the popularity of scientists in condensed matter and statistical physics is investigated [2]. The findings of this are quite different. *p(f)* decays exponentially, rather than in power-law fashion:

$$p(f) \sim e^{-\eta f}\;; \quad \eta \approx 0.00102.$$

Another popularity measure proposed is the frequency of appearance in the news [3].

Popularity of a web page is another context that is crucial for search engines. The quality of a search engine is to bring the "right" pages, in the "right" order. Although there may be many web pages which covers the topic, some web pages become more famous then others. Search engines use page ranking algorithms in order to sort the pages. Google's page ranking algorithm is based on number of in-links to page and the rank of the pages that initiate the link [4]. Intuitively, if a page gets more links, it has to have more "value". Similarly, if a page gets a link from an important page, that is highly ranked page, it's "value" should be increased. Web page linking to another page forms a *network of web pages*.

Incrementing the value of someone based on the reference of a valuable one is the trust mechanism [5]. Agent A trusts agent B. Since agent B trusts agent C, agent A also trusts agent C. In this way a *trust network* is constructed.

## 1.2. Social Networks

Social networks are difficult to study since data collection is difficult. One of the early studies is done by Milgram which lead to *six degrees of separation* principle [6]. The basic idea is to send a letter from a source person to a destination person. The rule is that anybody who receives the letter should send it to somebody that he knows by name. This experiment establishes *who-knows-who* network and concluded that any two persons are connected by a chain of persons whose length is around 6 which is much smaller then expected. Watts and Strogotz improved this concept to "small-world" [7].

Co-authorship of scientific papers is another social network. Power-law is observed in degree distribution of these networks. Power-law degree distribution,

$$p(k) \sim k^{-\gamma}$$

where $p(k)$ is the probability of having degree of $k$, seems to be quite common in many complex networks including internet router, www, e-mails, movie actors, co-authorship of scientific papers [8, 9, 10, 11, 12].

Citation of scientific papers forms a network in which power-law degree distribution is also valid [13]. Number of citations of a paper gets could be a measure of its popularity. Number of links a web page gets is another measure of fame. In who-know-who networks, the more links one gets, the more popular she becomes.

Another social network is studied in the co-occurrence in the news. Persons occur in the news are represented as vertices and there is an edge if two persons occurred in the same news article [14]. The more a person occurs in news, the more famous she becomes.

### 1.3. Motivation of the Model

A node should "know" the other node in order to link to it. In cases of social networks or complex networks, the number of nodes is, simply, too large. Therefore, no node should be able to "know" all the other nodes[1]. Only a small fraction of the total nodes can be known by any single node. On the other hand, there is no limit to be known. A node can be known by all of the nodes in the network.

Some one discovers a new web site and starts to recommend it to his contacts. A contact that receives the recommendation evaluates it. Sometimes keeps it. Sometimes he also recommends the site to others. This behavior is the basic idea of simple recommendation model that is developed in this paper.

### 1.4. Organization

The concept of "to know" is the center of this paper. This paper investigates the effect of the size of memory to the fame. A model is defined. Simulation results of different memory sizes, population sizes are investigated. The findings are interpreted. A model for fame is searched. The paper concluded by a conclusion and future work.

[1] Some proposed models for complex networks such as prefrential attachment requires information about mostly connected nodes which is a global information.

## 2. Definitions

*Population* is made out of persons. The *size* $\boldsymbol{n}$ of the population is defined to be the number of persons in the population. $\boldsymbol{p_i}$ is the i'th person where $1\leq i\leq n$. A person has a *memory* which is a list of persons. The *size* $\boldsymbol{m_i}$ of the memory of the person $p_i$ is the maximum number of persons that can be stored. The *memory ratio* $\boldsymbol{\rho_i}$ of the person $p_i$ is defined to be the ratio of the memory size to the population size, that is

$$\rho_i = m_i / n \qquad \textbf{(1)}$$

*Total memory capacity* of the population is the summation of the size of memories of the individual persons that is $n{\times}m$.

A person $p_1$ *knows* person $p_2$ if person $p_2$ is in the memory of person $p_1$. The *knownness* $\boldsymbol{k_i}$ of a person $p_i$ is the number of persons that "know" the person. If no body knows a person $p_i$, that is $k_i$=0, then person is called *completely unknown*.

The *fame* $\boldsymbol{f_i}$ of a person $p_i$ is defined as the ratio of the *knownness* to the population size, that is

$$f_i = k_i / n \qquad \textbf{(2)}$$

Notice that if no duplication is allowed, then $k_i\leq$ n. Therefore $0 \leq f_i \leq 1$, that is $f_i$ is normalized.

Since it is the percentage of the population that knows the person, fame is used as a measure of famousness. As the percentage increases, the person becomes more *famous*.

**Recommendation.** Person *learns* if he gets a new person in his memory. A person $p_1$ *remembers* a person $p_2$, if $p_1$ selects $p_2$ among persons stored in his memory. Person *forgets* if he removes a person from his memory.

Persons interact by exchanging recommendation. Person $p_1$ *recommends* person $p_2$ to person $p_3$. Steps of recommendation process are:

- $p_1$ remembers $p_2$
- $p_1$ gives $p_2$ to $p_3$
- $p_3$ learns $p_2$

Person $p_1$ is called the *giver*. Person $p_2$ is called the *taker*.

A few remarks are needed. It is important to note that learning, remembering and forgetting have implicit selection processes. A model has to specify these mechanisms. Although the model is constructed for "people-knows-people" case, it is valid for "people-knows-item" case for any item.

A person can only recommend persons that he knows. In the case of large memory size, $m>n$, one has enough memory to remember every individual in the population; hence, there is no need to "forget". Assuming that no duplication is allowed, $m=n$ is the boundary condition. If $m<n$, then person has to "forget" a person, in order to

"learn" a new person since there is not enough space in his memory. Therefore, for $m<n$, the taker of the recommendation exchange has to select some person, forgets him, obtains an empty slot, "learns" the new person using this slot.

## 3. A Simple Recommendation Model

A simple recommendation model, in which the memory size is constant $m$ for all individuals, can be built on these concepts. Initially, the memories of the persons are filled with persons selected randomly where duplications are allowed. Any two persons are expected to have almost the same knownness, hence every one have the same fame initially.

The details of the selections are defined as follows:

- The giver and the taker are selected randomly.
- The giver randomly selects a person from his memory as his recommendation. This is the "remembering" process.
- The taker checks if she already knows the recommended person. If she remembers the person, then there is no need to anything. If she does not remember the recommended person, a memory location has to be freed. She randomly selects a person from her memory and forgets the person. Then, she learns the recommended person by storing the person into this location.

## 4. Method

This simple recommendation model is implemented in Java where the parameters are $n$, $m$ and the number of reference exchanges.

Different combinations of values of n and $\rho$ are used including n=100, 1000, 10000 and $\rho$=0.5, 0.3, 0.2, 0.1, 0.05, 0.005, $\cdots$.

The number of reference exchanges is tried to be as large as possible. As $n$ increases, it has to increase, too. It is started from $10^6$ for $n$=100 to $10^{10}$ for $n$=10,000 which seems to be large enough.

10 simulation runs are made for different combinations of $n$ and $\rho$ values. The results of these runs are visualized by Excel and Matlab. In the figures, results of two simulations runs are presented so that the fluctuations can be seen.

Random selections required by the model are implemented by random() method of java.lang.Math which is a pseudo random number generator with a uniform distribution.

# 5 Observations

System gets its steady state quite fast as seen in Figure 1. Due to short run times of simulation and the ease of visualization $n$=100 is used for initial observations. Then for large values of $n$ similar simulations are done. The similar patterns are observed. Firstly, the effect of $\rho$ is investigated. For values of $\rho>1$, it is not interesting. So the cases of $\rho\leq 1$ are studied.

## 5.1. Case $\rho$=1

Initially simulations are done for a population of $n$=100 and $m$=100, hence the memory ration $\rho=m/n=1$.

The results of many simulations consistently produce similar patterns. In the Figure 2, *Pre1* is the initial state and *post1* is the state at the end of the simulation of reference exchanges. In order to compare the difference between different simulations, a second data set is also given. *Pre2* and *post2* are the initial and final data for another simulation run.

At any given time memory dumps of individuals provide information of about who-knows-who. The knownness of person $p_i$ is calculated by counting the persons that know person $p_i$. The knownness table which consists of persons and their knownnesses is obtained. For better visualization, the data is sorted in descending order in knownness and the graph in Figure 1 is obtained.

If everybody is of the same fame, the graph should be a horizontal line. If there are minor fluctuations in the fame, then the difference in fames of the most famous person and the least famous persons should be minor. Since the data is sorted in descending order, a small negative slope should be expected.

The initial loading of the memory has some fluctuations, (range 73-128 in pre1). For $m=n$, the recommendation exchange flattens the fluctuations by removing duplicate entries in the memories. This is expected behavior since during the initial memory loading duplications are allowed but during the recommendation exchanges duplication prohibited.

## 5.2. Case ρ<1

An unexpected pattern forms when the memory ratio decreases. Almost uniform distribution of fame that is observed at $\rho=1$ disappears. Some people become more known where as some become less known. Further decreasing $\rho$ causes some people to completely forgotten by the population. Figure 3 gives the effect of $\rho$ to the change in fame.

For $\rho=0.5$ the range of knownness enlarges. Some people become more known then others. Everyone is known in varying degrees and there is no completely forgotten person.

Around $\rho=0.3$, some people become *completely unknown* by the community, that is no one knows the person.

It is important to notice that if a person becomes completely unknown, then there is no way for her to become known. Therefore to become completely forgotten is a critical sink state in terms of fame. The number of completely forgotten people increases as $\rho$ decreases as seen in Table 1.

### 5.3. Case ρ《 1

An individual who can remember only one person is an extreme case. This corresponds to $m$=1. As $\rho$ keeps decreasing, it approaches to this extreme value.

For $n$=100 and $\rho=0.01$ only one person is known by the population and the rest is unknown. This can be explained by the small size of the population. In Figure 1, where $n=10^3$, $m$=10, $\rho$=0.001, there are around 50 known people. Since $n$=100, $\rho$=0.01 means the memory of the persons have only one slot. Suppose this one slot is slightly dominated by a person $p_i$. Then $p_i$ would be recommended more by the recommendation process. Due to the rules of recommendation process, $p_i$ replaces the position of other people. This further increases its dominance. As the knownness of $p_i$ increases, it is recommended more. As the process continues, other persons would be removed from the memory of the population quite rapidly due to this positive feedback.

### 5.4. Distribution of Fame

Individuals have different fames in the population. One property that needs to be investigated is the distribution of fame.

Figure 4 gives the distribution of knownness with respect to $\rho$. The distribution of initial memory is bell-shaped as expected. The shape of the initial distribution does not change with $\rho$ except its mean decreases with the memory size.

The distribution of knownness shows some pattern with respect to changes in $\rho$. In the range of $\rho$=1.00 to $\rho$=0.50 both pre and post data are bell-shaped. Around $\rho$=0.30, an unexpected pattern emerges. Completely unknown persons appear. This pattern grows as $\rho$ decreases to 0.

The interpretation of the growing component near 0 is that less famous people in the population are increasing. The value at 1 means the number of individuals known by only one person. The value at 0 gives the number of completely unknown persons, the individuals that are completely forgotten by the society.

### 5.5. Change of Fame

Minimum value of fame is 0, when a person is completely unknown. As $\rho$ decreases from 0.5 to 0, completely unknown peoples start to appear. The number of completely unknown is keeps increasing as $\rho$ approaches to 0.

Maximum value of fame has an interesting behavior that deserves an explanation. The maximum fame slowly decreases as $\rho$ goes from 1.00 to 0.10. It reaches a minimum around $\rho$=0.10. Then it rapidly increases as $\rho$ approaches from 0.10 to 0 as seen in Figure 5.

This pattern can be explained. When $\rho$=1, that is $m$=100, everybody is known by everybody else so the fame is 1. As $\rho$ decreases, the memory of the individuals decreases. Since no one dominates the memories yet, people are almost evenly distributed in the memories. So the reduction of the maximum fame is due to the decrease of the memory size. But as $\rho$ keeps decreasing, after a certain point some people become completely forgotten and some others become the dominated ones. As $\rho$ approaches to the limit of 0, the more people are completely forgotten and a few people dominate the memories. Those that dominate take all the references. So the rapid increase of maximum fame can be explained due to this positive feedback.

### 5.6. Effect of Population Size

So far $n$=100 is considered. 100 is a too small size compared to complex networks that are considered for power-law or small-world properties.

For larger values of $n$, similar observations are obtained. Figure 6 gives the patterns for various values of $\rho$ for $n=10^3$. Table.2 also gives this interesting observation for $n=10^3$.

As in the case of $n$=100, the distribution of post data for $n=10^3$ starts with a bell-shaped curve. As $\rho$ decreases, a component near 0 increases from 376 (38%) for $\rho$=0.05, to 958 (96%) for $\rho$=0.005. Figure 7 compares $\rho$=0.05 for $n=10^2$, $10^3$ and $10^4$.

## 6. Conclusion

The effect of the memory size $m$ of individuals with respect to population size $n$ is investigated. The ratio $\rho$ of memory size to population size is used as parameter. A simple recommendation model which changes the memory of the individuals is defined.

The value of $\rho$ is changed from 1 to 0. For $\rho$=1, everybody is known by everybody else. For values of $\rho$ around 1, an individual is known by majority of the population. As $\rho$ decreases, some individuals become unknown by the population. As $\rho$ gets close to 0, almost everybody becomes unknown. On the other hand, as $\rho$ decreases some people become more known by the population. As $\rho$ gets close to 0, very few individuals become very well known.

Although in the model items stored in the memory were again people, the model is valid if some other items are stored, too. For example web pages or dentists could be the items to store. Consider persons in a country, web pages, scientific papers, radio stations, books. For all these practical situations the size of the items is much larger then the size of the memory of the individuals. Therefore, $\rho$ values close to 0 are realistic values.

The pattern of fame, that is observed around $\rho$=0 in the model has corresponding counterparts in these real life cases. Very few people are famous, where as there are millions known by very few. Same is true for web pages. Google is known by almost every internet user. Nobody knows the number of unknown web pages, since only a percentage of the entire web is cataloged by the search engines.

Recommendation could be the machinery of fame. This model could be the model of fame or the dynamics of population memory.

The rapid forgetting mechanism around $\rho$=0 is an interesting phenomenon to look for. Once an item has small knownness, then its knownness converges to 0 quite rapidly. This can be a model of disappearing of cultural values such as languages, traditions, piece of music, a poem or the extinction of a species once the numbers become few.

One more observation is that although the memory size of the individual is small, the total memory of the population, that is, the number of items in the memory of the population is much larger. On the other hand, population memory is less than the total memory capacity of the population that is $n \times m$.

## 7. Future Work

It is assumed that the memory size is the same for everybody. This assumption can be relaxed to lead variation of memory sizes. For example 10% of the population has larger memory then the rest of the population.

Initially everybody has almost the same popularity. This can also be relaxed by favoring group of people to be famous initially.

"How to become popular" is another related question. Given that the population has already famous people, how do new comers become famous, that is what percentage of the population should know the person? Since to make people learn somebody is done through advertisement, this could lead to a model for advertisement campaigns.

*Who-knows-who* information can be represented as a directed graph called *who-knows-who graph* in which the persons are the *vertices* of the graph. There is an *arc* from person $p_i$ to person $p_j$ if person $p_i$ knows person $p_j$. In complex networks some properties such as scale free and small world are investigated. These properties of who-knows-who graphs obtained using the reference model is investigated in the upcoming paper [15].

The author would like to thank Albert Ali Salah for his help in matlab programming.

## References

1. Simkin M. V., Roychowdhury V. P.: Theory of Aces: Fame by chance or merit?, (2003) cond-mat/0310049.
2. Bagrow J. P., Rozenfeld H. D., Bollt E. M., Avraham D. B.: How Famous is a Scientist? - Famous to Those Who Know Us, (2004) cond-mat/0404515.
3. Bingol H.: The frequency of appearance in the news as a popularity measure, (in preparation).
4. Brin S., Page L.: The anatomy of a large-scale hypertextual Web search engine, Computer Networks and ISDN Systems (30), (1998) 107-117.
5. Yolum P., Singh M. P.: Engineering Self-Organizing Referral Networks for Trustworthy Services Selection, IEEE Transactions On Systems, Man, And Cybernetics, 3 (2004) 396–407.
6. Milgram, S.: The small-world problem, Psychology Today, **2**, (1967) 60-67.
7. Watts, J.D., Strogatz, S.H.: Collective dynamics of ‘small-world’ networks, Nature **393** (1998) 440-442
8. Newman M.E.J.: The structure and function of complex networks, (2003) cond-mat/0303516.
9. Dorogovtsev, S. N., Mendes, J. F. F.: Evolution of Networks, (2001) cond-mat/0106144v2.
10. Albert, R., Barabasi, A. L.: Statistical Mechanics of Complex Networks, In Reviews of Modern Physics **73** (2002) 47-97, (cond-mat/0106096).
11. Kirlidog, M. Bingol, H.: The shaping of an electronic list by its active members, ITIRA’03 (2003) 40-48.
12. Newman M.E.J.: The structure of scientific collaboration networks, PNAS **98** 2, (2001) 404-409.
13. Redner, S.: How Popular is Your Paper? An Empirical Study of the Citation Distribution, (1998) cond-mat/9804163.
14. Ozgur, A., Bingol H.: Social Network of Co-occurrence in News Articles, LNCS 3280 (2004) 688-695.
15. Bingol H.: Graph properties of who-knows-who graphs, (in preparation).

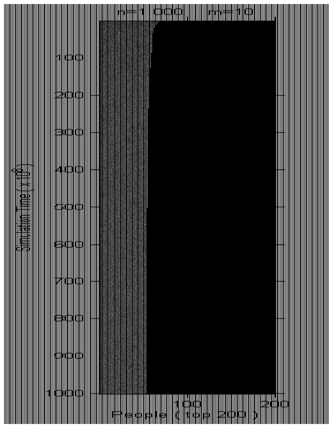


**Fig. 1.** For $\rho$=0.001 with $n$=$10^3$, $m$=10, $10^{11}$ recommendations are made. Time is on the y-axis with units of $10^8$ recommendations. Peoples are on x-axis sorted according to knownness. More active 200 is shown, since majority of the people become completely unknown at the very early stages of the simulation. The system settles down quite fast. Note that after 500x$10^8$ there is no change. Black is unknown and white is the most known.

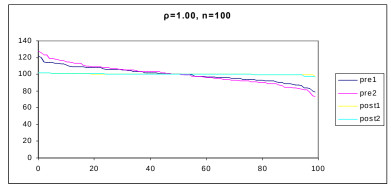


**Fig. 2.** This is the results of population of size $n$=100. All peoples are of memory size $m$=100. The individuals are on the x-axis and their knownness on the y-axis. Data is sorted in descending order.

In the graph, *pre1* and *pre2* are results of initial load of memories. *Post1* and *post2* are the results after $10^6$ reference exchanges.

There are minor fluctuations between the two simulation runs. For *pre1* and *pre2*, the range is 79-122 and 73-128, respectively. One expects that the upper bound of knownness should be $n$. Knownness higher then $n$, such as 122 or 128, mean some persons are known by more then once by a person since duplication is allowed in the initial loading.

The ranges of *post1* and *post2* are 98-102 and 97-102, respectively. It is observed that for $m$=$n$, fluctuations are removed, the range is reduced. This is expected since duplication is not allowed during recommendation exchange.

**Table 1.** Number of unknown people increases as $\rho$ goes to 0 for population of $n=100$. In the limit case of $\rho=0.01$ only one person is know and the rest of 99 people is unknown by the society.

| **n=100, ρ** | **1.00** | **0.50** | **0.30** | **0.20** | **0.10** | **0.05** | **0.01** |
|---|---|---|---|---|---|---|---|
| **# of unknown** | 0 | 0 | 7 | 34 | 79 | 88 | 99 |
| **#fame (max)** | 102 | 63 | 51 | 50 | 46 | 57 | 100 |

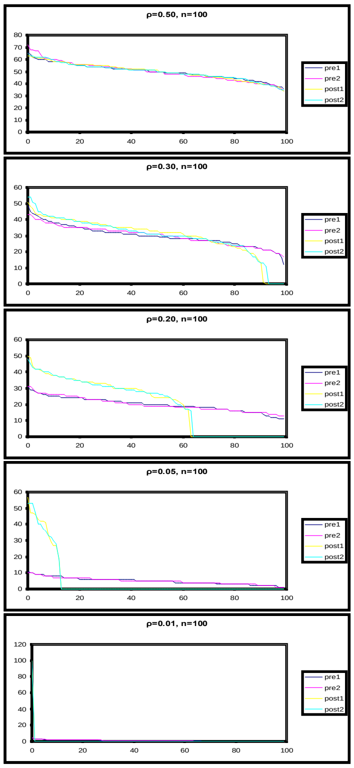


**Fig. 3.** Effect of memory ratio $\rho$ for $n$=100. As the memory ratio $\rho$ decreases, some people become famous in response to that, more and more people become unknown.

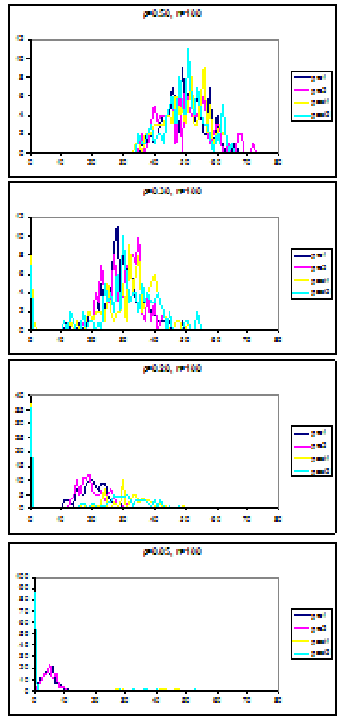

**Fig. 4.** Change of distribution of fame with respect to memory ratio $\rho$. As $\rho$ goes to 0, the distribution of fame becomes distorted. The mean of the initial memory moves to 0 since the memory size gets smaller. A few individuals becomes very famous, while majority of the population becomes more and more unknown by the population.

**Table 2.** Pattern near 0 as $\rho$ decreases for $n$=1,000.

| n= 1,000 ρ | 0.50 | 0.15 | 0.10 | 0.05 | 0.03 | 0.01 | 0.005 |
|---|---|---|---|---|---|---|---|
| # of unknown | 0 | 0 | 16 | 376 | 661 | 903 | 958 |
| % of unknown | 0.00 | 0.00 | 0.02 | 0.38 | 0.66 | 0.90 | 0.96 |

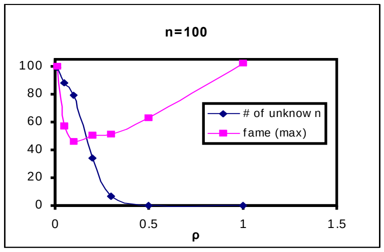


**Fig. 5.** As memory ratio decreases from 1 to 0, number unknown increases. Interestingly, maximum fame slowly decreases and then rapidly increases.

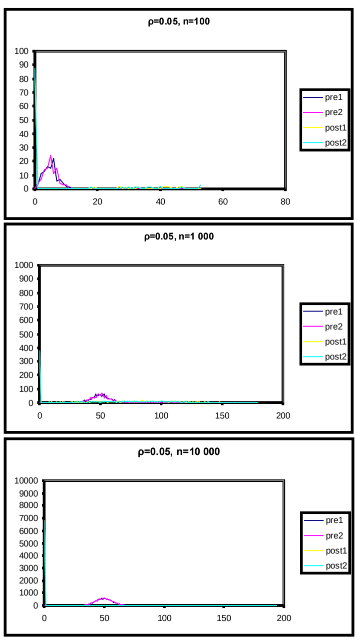


**Fig. 7.** Change of distribution of fame with respect to $n$ for $\rho$=0.05 for $n$=100, 1,000 and 10,000. Note that the component near 0 is increasing as $\rho$ decreases. Two data sets represent the same behavior.

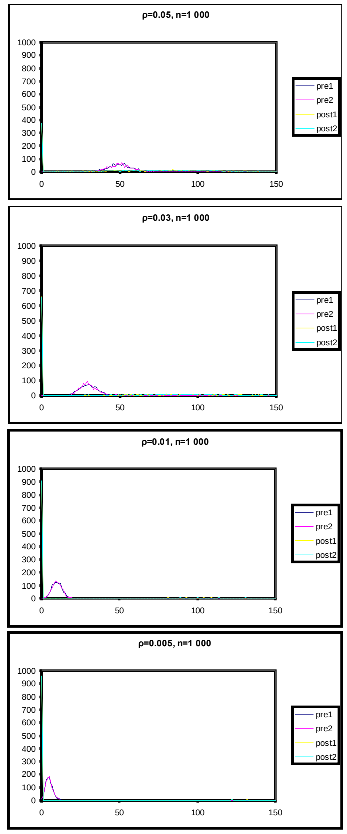


**Fig. 6.** Change of distribution of fame with respect to $\rho$ for $n$=1,000. Note that the component near 0 is increasing as $\rho$ decreases. Two data sets represent the same behavior.