A way to characterize complex cellular automata and those able to perform density classification

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In this work we consider particular mathematical models useful in studying complexity: the cellular automata (CA) [1]. They are constituted by a great number of connected entities which interact at a local level, showing, at a global level, emergent behaviors and self-organization. CA were introduced by von Neumann and Ulam as simple models for the study of some biological processes. CA have been used as models for systems like bugs' colonies, immune systems, economic systems, etc., because they show the emergent computation and complex behavior typical of these systems.
A CA is a discreet dynamical system in which the space is represented by a n-dimensional regular grid. Each element of the grid (called cell) can be considered as the basic element of CA and it contains a datum a_i^t that represents the state of the i^{th} cell at time t. A CA configuration (c_t) is the set of the states in which all the cells are at a particular time t. The state of each cell evolves following a rule that depends both on the state of the cell itself and on the state of a certain number of cells that compose the neighborhood of the cell. The behavior or the evolution of a CA, starting from an assigned initial configuration, is the series of configurations $\{c_t\}_{t\geq 0}$. CA can show different types of behavior: periodic, chaotic and complex. The most interesting one is the complex behavior, in which self-organized complex structures interacting with each other emerge in the evolution. The problem of finding this kind of CA among all the possible rules, is not so simple. Once the number of states (k) and the number of cells that compose the neighborhood (m) are fixed, the number of all the possible rules is k^{k^m}. For one-dimensional CA with $k=2$ and $m=3$ (elementary CA) it is possible to do an exhaustive examination of all the rules. Nevertheless, by increasing k and m, the high power of the rules' space make this examination difficult to be undertaken. For this reason, the use of genetic algorithms (GA) can be viewed as an interesting alternative approach. Various experimentations in this direction have been done [2], [3], [4].
The genetic algorithms, introduced by John Holland [5], are inspired by the natural evolution. They are used a lot in the context of evolutionary computation, particularly for the optimization and the automatic learning. The main idea is to evolve a population of candidate solutions for a particular problem using some operators (mutations and cross over) that are inspired to genetics and natural selection.
In this work the genetic algorithms have been used for exploring the rules' space of elementary CA and multi-states CA ($k>2$) with the aim to characterize, by different fitness functions, the rules that show complex behaviors and the rules able to perform density classification [3], [4].
The density classification problem consists in determining some rules that evolve towards an homogeneous final configuration constituted by the state with the higher concentration in the initial configuration.
Rules have been evolved using GA. The input entropy, able to select complex rules [2], was used as fitness function in some experiments. In other experiments, another fitness function [4] was specifically chosen in order to select rules which perform density classification. The evolved rules have been subsequently evaluated by computing, for each rule, some parameters taken from the literature [6] by broadening their applicability to multi-state CA.
Finally, some considerations have been drawn about the relationships between the values of the parameters described above and the specific characteristics of the analyzed rules.

References

[1] Wolfram S: Universality and complexity in cellular automata. Physica D 1984; 10: 1-35.
[2] Bilotta E, Lafusa A, Pantano P: Searching for complex CA rules with GAs. Complexity 2003;8(3):56-57.
[3] Packard NH: Adaptation toward the edge of chaos; in Kelso JAS, Mandell AJ, and Shlesinger MF (eds): Dynamic Patterns in Complex Systems. World Scientific, 1988, pp 293-301.
[4] Mitchell M, Crutchfield JP and Hraber PT: Evolving Cellular Automata to Perform Computations: Mechanisms and Impediments. Physica D 1994; 75: 361-391.
[5] Holland JH: Adaptation in Natural and Artificial Systems, ed 2, MIT Press, 1992.
[6] Oliveira GMB, de Oliveira PPB, Omar N: Definition and Application of a Five-Parameter Characterization of One-Dimensional Cellular Automata Rule Space. Artificial Life 2001; 7(3): 277-301.