Optimization and control of the urban spatial dynamics

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Abstract

The urban planning concerns the assignment of a land use to each land cell. This process coexists and may conflict with the complex self-organized dynamics of the urban system, which should be constrained by the plan. The purpose of this study is the identification of a method for attaining the planned goals through the utilization of the self-organized dynamics, and the minimization of the constraints. The paper is organized in three steps. First: the urban plan is defined as a process of optimization. Second: the set of optimal solutions is compared with the configurations resulting by the self-organized dynamics. Third: a method for the convergence of the self-organized dynamics with the optimal configuration is proposed. In conclusion the study shows that planning a complex system may be an hard task, while the control and the utilization of the self-organized dynamics helps in the attainment of a total utility.

Introduction

Two main streams of problems arise for urban strategic planning from the widely recognized self-organizing character of the urban dynamics [1]. From one side the chaotic behavior highly dependent on initial conditions of the self-organizing system makes quite unpredictable the effects of the planning policy [2]. On the other side the urban planning is questioned by the ability of the urban system to steer itself [3]. While classic urban planning seeks to regulate the urban structure with a top-down approach, this last evolves with the interplay of a lot of local actions[4]. Thereby these two processes, even if coexistent in a city, may conflict. One solution to this topical interest problem in urban planning is a just-in-time method, as opposed to a just-in-case, where projects are delimited in both space and time and coupled with a constant reevaluation of the whole sketch [5].

In this paper we choose a different way. We suggest the utilization of this self-organizing character in order to achieve in an easier way the planning objectives. The proposed method includes: first the definition of the plan as an optimization process, second a comparison of the optimal solutions with the configurations emerging from the self-organized dynamics, and third the convergence of the self-organized dynamics with the optimal configuration. In a first step this method is applied to a system with two land uses, and in the second step a more realistic situation is utilized in order to show the possible utilization of the proposed method.

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Optimization

Even if urban planning is a complex process involving many actors which bargain for urban projects, after all it ends up in the assignment of land uses or activities to land parcels, so that the public utility, however defined, is also attained. The problem of the optimal spatial assignment of activities was formulated by Koopmans and Beckman [6]. Later the optimization process has been proposed in the context of rational urban planning [7][8], as well as in architectonic design [9]. Recently new methods for the research of the global optimum such as genetic algorithm, have been utilized in order to establish the set of efficient alternatives in a multi-objective optimization [10],[11]. This multi-objective optimization method refers to a theory of the urban planning in which a set of efficient alternatives, or Pareto optima, are proposed to the decision makers. In other words the optimization method is utilized as an aid to the decision process, and the efficient solutions are taken as alternatives to be bargained among the decision makers[12].

In order to analyze the relation between the urban plan and the self-organized dynamics, we consider firstly the plan as an optimization process. The land-use planning is an assignment of land uses to land parcells in order a total utility function in which the spatial relations among land-uses are included, is maximized. To formalize the problem, let us suppose a set of possible land uses or states of a cell i of a squared grid having n cells in each side. Hence $s_i = \{h, r, ...\}$, the state of cell i, represents the land use assigned to a cell i (for instance housing, retail etc.) and N_h is the total number of cells in state h which is established as a constraint to the optimization. In addition, for each cell i is established a neighborhood Φ_i, and an utility u_{hi} of land use h in cell i which depends on h, and on the land uses localized in the Φ_i neighborhood, as in the following equation:

$$u_{hi} = f(h, s_{\Phi_i}), \tag{1}$$

where u_{hi} is the utility of land use h in cell i, and f is a function. This function may take many forms. In this paper it has been chosen the simplest, i.e. a linear combination, which simulates a perfect substitution of factors. Other functions with imperfect substitution of factors giving different results, are not analyzed in this paper. According to the linear combination method, the utility is calculated as in the following expression:

$$u_{hi} = \sum_r \left(m_{hr} \sum_{j \in \Phi_i} r_j \right), \tag{2}$$

where $r_j = 1$ if the use r is assigned to cell j, otherwise $r_j = 0$. Further m_{hr}, an element of the matrix $\mathbf{M}$, is the utility for use h to be surrounded by use r $(\sum_r m_{hr} = 1)$. In words, equation (2) states that utility depends on the quantity of the land uses in the neighborhood multiplied by a parameter. The total utility of each land use is:

$$U_h = \sum_i u_{hi}. \tag{3}$$

Moreover, a weight is assigned to each land use in order to calculate the total utility U_T, as in the following equation:

$$U_T = \sum_h W_h U_h. \tag{4}$$

This W_h weight $(\sum_h W_h = 1)$ represents the economic capacity of the land use h to bid for the land, or the importance assigned to this land use in case a plan is implemented. This is an usual assumption in multi-criteria decision making, and it is the simplest method to compare the otherwise incommensurable vectors of the utilities. The planning problem is stated as the research of the assignment which maximizes the total utility, for each possible set of weights. By using this method, under the assumption in the previous equation (4), one obtains the set of Pareto optimal solutions[13]. For each of these solutions it is impossible to improve the utility of a land use U_h without decreasing the utility of another land use U_r.

In the following experiments the maximization of utility is obtained by using the simulated annealing method[14]. Not only this is a general optimization method which can be applied to each sort of problems, but it is formulated as a simulation of a dynamic process which can be easily compared with the self-organized dynamics. When the simulated annealing method is applied, the process begins by using a random pattern of land uses. Then, at each step one couple of cells is taken at random, the land uses are exchanged, and the total utility U_T of the new pattern is evaluated by using equation (4). A new configuration is considered or not, according to the method of simulated annealing which decreases the temperature thus allowing the system to reach and maintain a stable state in which the energy $E = -U_T$ is minimized, and the total utility is maximized.

Nevertheless, the urban system is usually able to reach some stable state by using its self-organizing character. In this case the maximization of the total utility is not a necessary, even if a possible [15] outcome of the process. We call self-organized dynamics this process, which is presented in the following section.

Self-organized dynamics

A lot of studies in the micro-simulation of the urban dynamics are available. Usually a cellular automaton framework is utilized, even if an agent based modeling approach is emerging as a novel method[16]. For the most part these approaches consider an expanding system from a central seed. Since we are more interested in the internal rearrangement of the system then in its growth, a different approach is utilized which is partially similar to that utilized in the simulated annealing method. In fact beginning from a random pattern of the established land uses, at each step two cells are chosen at random and the land uses are exchanged if this exchange does not decrease the sum of the utilities of the two land uses. Thereby the land use h in cell i is exchanged with the land use r in cell j if:

$$u_{ri} + u_{hj} > u_{hi} + u_{rj}. \tag{5}$$

This method simulates a spatial dynamics in which the local individual utility is maximized, as in a market in which a transaction happens only if a couple of individuals thinks that they will be more satisfied after, than before the transaction. In addition this method mimics the efforts for establishing a solution through a lot of repeated trials, like in the simulated annealing method. But, in this case at each exchange, is maximized the sum of the local utilities instead of the total utility U_T which may both increase or decrease because the utility of the surrounding cells is not considered. The stability of this self-organized dynamics is attained when for each couple of cells the exchange does not increase the sum of the utilities of the two land uses.

The Utopian situation corresponds to the similarity between the optimal configuration and the result of the self-organized dynamics. In this case the utilities obtained with the optimization process and with the self-organized dynamics are equivalent in the steady state. This comparison allows to divide the set of Pareto solution in two subset: the A-set of solutions in which the optimum is similar to the outcome of the self-organized dynamics and the B-set for which there is no similarity. This similarity is evaluated comparing the utility attained by the optimal configuration in relation with the utility attained by the self-organized dynamics. Thereby an index of dissimilarity I^d is defined:

$$I^d = \frac{U_T^1 - U_T^2}{U_T^1}, \tag{6}$$

where U_T^1 is the total utility attained with the optimal configuration and U_T^2 is the total utility attained with the self-organized dynamics. This index ranges from zero, because $U_T^1 \geq U_T^2$, to 1.

Convergence of the self-organized dynamics with the optimal configuration

In case the optimal solution and the self-organized dynamics do not match, one would like to make the self-organized dynamics convergent with the optimal configuration. The classic urban planning approaches this problem by establishing a feedback for each point of the urban surface. In case the land use in an urban zone does not agree with the established use, and this mismatch is officially observed by the control agency, then a sort of penalty, usually established by the law, is applied in order to reestablish the planned assignment. This is the principle of the dynamic system control, in which there is a master system, the plan, and a slave system, the urban dynamics, and each point of the master is coupled with the corresponding point of the slave system. This method simply constraints the self-organized dynamics into the optimal configuration. The energy spent in this process by the control agency depends on the *distance* between the urban plan and the result of the self-organized dynamics. For this reason often an urban plan is conceived in a way that this distance is reduced in order to make the desired configuration more attainable.

In opposition to the classic urban planning, the proposed method is based on the utilization of the self-organizing characteristic of the urban system and on the minimum number of cells whose established land use is forbidden to change during the simulated dynamics. These pinning cells play the role of control but also of catalysts accelerating positive effects in the urban dynamics [17] without itself being transformed. The identification of the minimum number of pinning cells which is able to make the self-organized dynamics convergent with the optimal configuration is the core of the problem. This aspect has been widely studied in the field of the control of chaotic spatio-temporal systems [18], [19], and in essence consists in the identification of the cells which are *strategic* for the control of other cells in the spatial dynamics.

To establish the minimum number of cells able to control the urban dynamics, these are sorted according to the index of influence. The method utilized to calculate the index of influence of a cell is based on a kind of input-output analysis. In fact in the optimal configuration each cell generates and receives utility from the bordering cells. According with equations (2) and (4), this utility v_{ij} generated by the land use r in cell i in relation to the land use h in cell j is defined in the following way:

$$v_{ij} = W_h m_{hr}. \tag{7}$$

By using the previous equation, the matrix $\mathbf{G}$ of the exchanged utility is calculated. Each element g_{ij} of the matrix is the ratio: utility generated by the cell i in the cell j–total utility generated in cell j by the whole set of cells, as in the following expression:

$$g_{ij} = \frac{v_{ij}}{\sum_k v_{kj}}. \tag{8}$$

In order to consider both the direct and the indirect effects the following matrix is calculated:

$$\mathbf{G}^\star = \mathbf{G} + \mathbf{G}^2 + \mathbf{G}^3 + \dots \tag{9}$$

The index of influence I^i for each cell is then calculated as the difference between the capacity to influence ($\sum_j g^\star_{kj}$) minus the degree of being influenced ($\sum_i g^\star_{ik}$):

$$I^i_k = \sum_j g^\star_{kj} - \sum_i g^\star_{ik}. \tag{10}$$

In the first step t of the ordering process, the first pinning cell k is chosen which corresponds to the maximum of I^i. Than we reason in the following way. If the cell k is established as a pinning site, the control of the other cells immediately dependent by k cell is useless. We would rather that the second pinning cell has an high index of influence, while being the less dependent on the first. For this reason we subtract to the capacity of control of each cell l a share, $g^\star_{kl}/\sum_i g^\star_{il}$, corresponding to the share of control exercised by the k cell on cell l. Thereby in the next step $t+1$ the elements of the matrix $\mathbf{G}^\star$ are recalculated in the following way:

$$g^\star_{lj}(t+1) = g^\star_{lj}(t)\left[1 - \frac{g^\star_{kl}(t)}{\sum_i g^\star_{il}(t)}\right]. \tag{11}$$

$$\text{matrix } \mathbf{G} \; (\textstyle\sum_i g_{ij} = 1)$$

$$\mathbf{G}^\star = \mathbf{G} + \mathbf{G}^2 + \mathbf{G}^3 + \ldots$$

$$k = \max(I_k^i = \textstyle\sum_j g_{kj}^\star - \sum_i g_{ik}^\star)$$

$$g_{lj}^\star(t+1) = g_{lj}^\star(t) \left[1 - \frac{g_{kl}^\star(t)}{\sum_i g_{il}^\star(t)} \right]$$

$$\forall_j g_{kj}^\star = 0$$

Figure 1. The algorithm for ranking the pinning cells.

Further the row k is set equal zero in order to exclude the cell k to be newly selected as a pinning cell, and the process is repeated until for each cell, the index related to its capacity to influence the land use of the other cells is calculated (figure 1).

Clearly a relation should exist between the number of pinning cells and the similarity between the optimal configuration and that obtained with the self-organized dynamics. In order to analyze this relation, we define D_{pq} as the distance between the configurations p, the plan, and q, the outcome of the self-organized dynamics. This distance is calculated by summing up the cells having a different land use in p and q. By using this distance we consider the following relation:

$$D_{pq} = f(C_q), \tag{12}$$

where C_q is the number of pinning cells, successively taken from the ranked list established by using the previous explained method. Usually D_{pq} decreases with the increase of C_q. In fact in case all the cells are declared as pinning cells, this is like in the classical urban planning, and $D_{pq} = 0$. In turn we can decide the quantity of pinning cells, beginning from the first in the ranked list and evaluate the rate at which the distance decreases, looking for the less number of pinning cells resulting in an acceptable distance. In order to study this and the other aspects in the next section some theoretic experiments are shown which highlight, with an increasing complexity, the relation between optimization and control in the spatial dynamics.

Experiments and discussion

In this section the proposed method is applied to a set of theoretic cases of increasing complexity. In each case, through the variation of the weights the set of optimal solution is obtained. Further the relationship between this set and the corresponding results of the self-organized dynamics is discussed in order to highlight the conditions under which the control of the complex system is the easiest and the most efficient.

In order to study the simplest situation it has been chosen a 10×10 lattice surface and two possible land uses for each cell: the housing (h) and retail (r) land use. In the first three experiments, the total quantities of each land use are constrained to 50. The rules of interaction refer to three paradigmatic cases. *Integration*: each land use is attracted by the other land use. *Segregation*: each land use is attracted by a similar land use. And *integration and segregation*: the housing land use is attracted by the retail land use, while this last is attracted only by itself. The neighborhood Φ_i is limited to the eight bordering cells. The corresponding values s of the elements of the $\mathbf{M}$ matrix are shown in table 1.

The segregation dynamics, has been widely investigated, beginning with the work of Schelling [20], with the purpose of connecting the emergent properties of the resulting pattern with a parameter related

Table 1. Matrix $\mathbf{M}$ in the three cases.

Integration			Segregation			Integration-Segregation		
Land use	Hous.	Ret.	Land use	Hous.	Ret.	Land use	Hous.	Ret.
Housing	0	1	Housing	1	0	Housing	0	1
Retail	1	0	Retail	0	1	Retail	0	1

to the degree of segregation. The self-organized dynamics here presented as a first step in the theoretic experiments, is very similar to these previous models. In fact in the first two sets of experiments (*integration* and *segregation*) the two extreme cases with a low and an high degree of segregation are considered. In the third experiment (*integration and segregation*), through the variation of the weights assigned to the two land uses (one of which is totally devoted to integration and the other one to segregation) we explore the different configuration emerging, as from a variation of the degree of segregation. In turn the results of the optimization process may differ from the Schelling model, because the total utility is considered and not only that of the two exchanging land uses.

Integration

In the *integration* experiment, for each set of weights the outcome of the optimization is similar to that obtained with the self-organized dynamics (figure 2), and practically all the optimal configurations belong to the A-set. The resulting total utilities for each set of weights are also similar (figure 3). This effect depends by the equal number of land uses. In fact, because the utility is generated by a couple of different cells, the number of housing cells surrounding a retail cell in the steady state is equal to the number of retail cells surrounding an housing cell. A set of rows horizontally or vertically disposed of alternate land uses, is the configuration that attains this effect, and the utility for each land use is always the same. The Pareto front is in fact represented by only one point, and the optimal configuration roughly coincides with the result of the self-organized dynamics. This is the Utopian situation in which the social utility agrees with the individual utility and in essence a plan is not necessary.

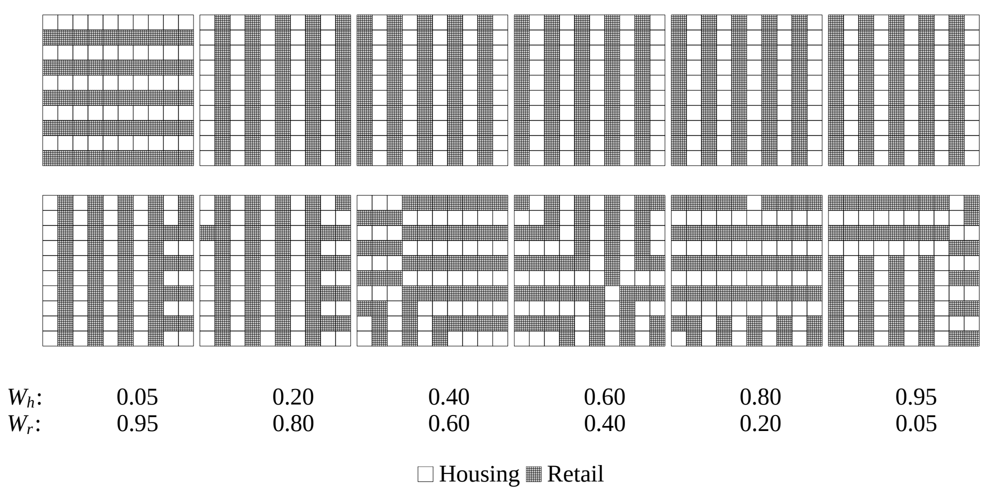


Figure 2. Integration. First row. The patterns obtained through optimization. Second row. The patterns obtained by applying the rules of the self-organized dynamics to a random pattern. At the bottom of each column the set of weights utilized for housing (W_h), and retail (W_r) is shown.

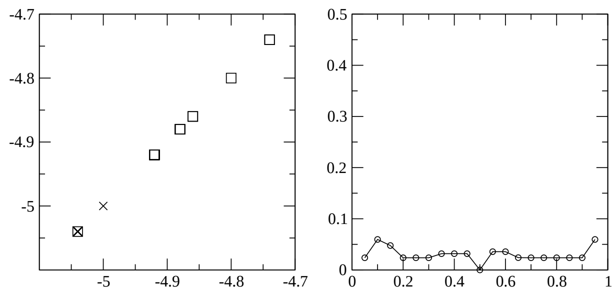


Figure 3. Integration. Left side, the Pareto front. Cross: configurations obtained through optimization. Square: configurations obtained by applying the rules of the self-organized dynamics to a random pattern. X axis: average utility of housing (negative). Y axis: average utility of retail (negative). Right side: the variation of I^d, the index of dissimilarity, in relation to W_h, the weight applied to the housing. X axis: W_h, Y axis: I^d.

Segregation

In the *segregation* experiment the patterns emerging from the applying of the segregation rules are twofold: the concentration of one use in the center and the division of space (horizontally or vertically) (figure 4). In fact due to the finite size of the surface, it is not possible for the two land uses, at the same time, to be circular shaped. The Pareto front is thereby represented by three points: the first two corresponding to the concentration of one use in the center and the third corresponding to the the division(figure 5). When the self-organized dynamics method is applied it produces a pattern which is similar to that produced by the optimization process (figure 4): all the optimal configurations belong to the A-set. Even if the utilities of the two land use are in opposition the situation is similar to the previous one and in essence a plan is superfluous.

Figure 4. Segregation. First row. The patterns obtained through optimization. Second row. The patterns obtained by applying the rules of the self-organized dynamics to a random pattern. At the bottom of each column the set of weights utilized for housing (W_h), and retail (W_r) is shown.

Integration and segregation

The results of *integration and segregation* experiment appear similar to those of the two previous experiments: both the concentration of one use in the center and the rows horizontally or vertically disposed of

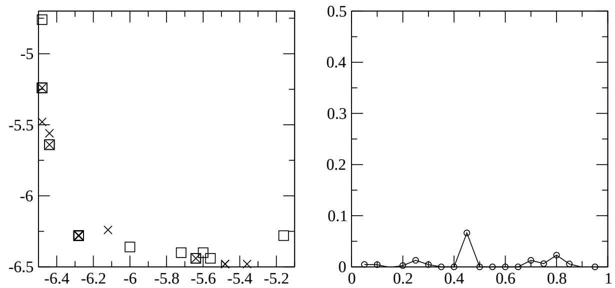


Figure 5. Segregation. Left side, the Pareto front. Cross: configurations obtained through optimization. Square: configurations obtained by applying the rules of the self-organized dynamics to a random pattern. X axis: average utility of housing (negative). Y axis: average utility of retail (negative). Right side: the variation of I^d, the index of dissimilarity, in relation to W_h, the weight applied to the housing. X axis: W_h, Y axis: I^d.

alternate land uses (figure 6). In order to compare more precisely these sets of experiments, let us focus on the differences in the matrices $\mathbf{M}$. This matrix is a-symmetric in this experiment while it was symmetric in the two previous ones. However this difference is only apparent. In fact, according with equation (7), the total utility U_T can also be calculated as the sum of the utilities related to each couple of bordering cells:

$$u^{\star}_{ij,hr} = W_h m_{hr} + W_r m_{rh}, \tag{13}$$

where $u^{\star}_{ij,hr}$ is the utility related to the couple of bordering cells i, where the land use h is located, and j, where the land use r is located, and $U_T = \sum_{ij} u^{\star}_{ij}$. Further the symmetric matrix of interaction $\mathbf{M}^s$ is defined in which:

$$m^s_{hr} = m^s_{rh} = \frac{W_h m_{hr} + W_r m_{rh}}{2}. \tag{14}$$

The total utility can also be calculated by using this symmetric matrix, as in the following equations:

$$u^{\star}_{ij,hr} = 2m^s_{hr}, \text{ and } u^{\star}_{ji,rh} = 2m^s_{rh}. \tag{15}$$

Because each couple is considered twice the total utility calculated with this method equals that calculated with equation (4). Now consider the symmetric interaction matrices $\mathbf{M}^s$ of the *integration* and of the *integration and segregation* experiments in relation to the set of weights: $W_h = 0.95$ and $W_r = 0.05$. As it is straightforward from the first part of table 2 these two matrices are similar as well as the interaction matrices of the *segregation* experiment and of the *integration and segregation* in relation to the set of weights: $W_h = 0.05$ and $W_r = 0.95$ (second part of table 2). This is why the pattern (figure 6) obtained

Table 2. Comparison of the matrices $\mathbf{M}^s$.

$W_h = 0.95$ and $W_r = 0.05$

	Integration		Integr.-Segreg.	
Land use	Hous.	Ret.	Hous.	Ret.
Housing	0	0.5	0	0.475
Retail	0.5	0	0.475	0.05

$W_h = 0.05$ and $W_r = 0.95$

	Segregation		Integr.-Segreg.	
Land use	Hous.	Ret.	Hous.	Ret.
Housing	0.05	0	0	0.025
Retail	0	0.95	0.025	0.95

with $W_h = 0.05$ and $W_r = 0.95$ is similar to that obtained in the *segregation* experiment, while the pattern obtained with $W_h = 0.95$ and $W_r = 0.05$ is similar to that obtained with the *integration* experiment. In addition an intermediate pattern (a transition between the two) is obtained when the weigths assigend to the land uses are similar.

The Pareto front is less convex than in the previous experiment, while the outcomes of the self-organized dynamics and of the optimization process are similar (A-set) till the weight assigned to the retail is greater than that assigned to housing (figure 7). Especially in the intermediate regime and when the weight assigned to the housing is greater than that assigned to retail, the optimal solutions belong to the B-set.

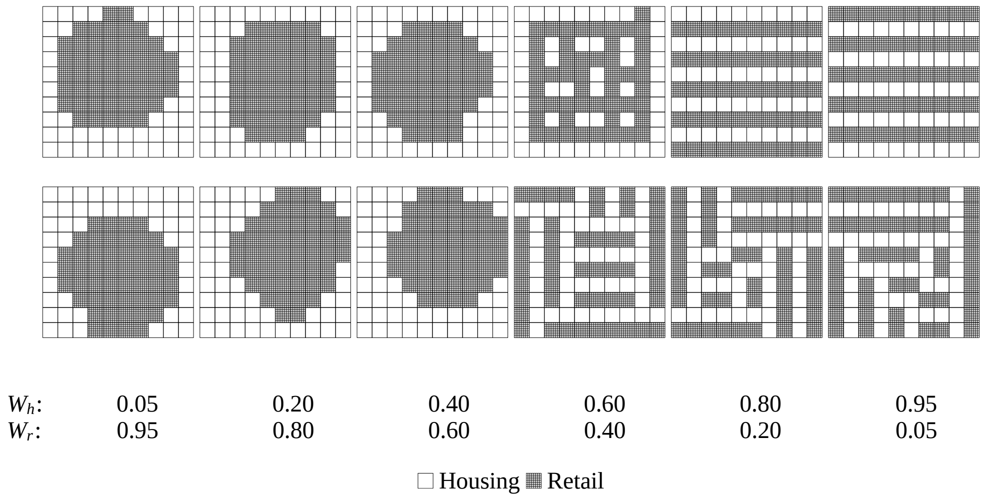


Figure 6. Integration and segregation. First row. The patterns obtained through optimization. Second row. The patterns obtained by applying the rules of the self-organized dynamics to a random pattern. At the bottom of each column the set of weights utilized for housing (W_h), and retail (W_r) is shown.

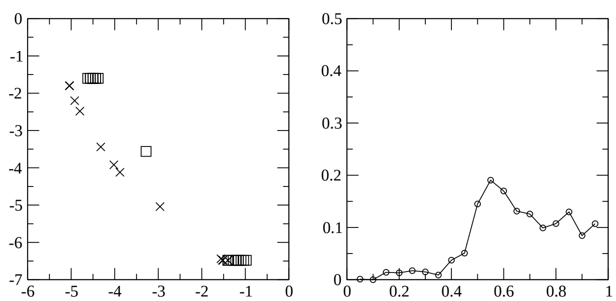
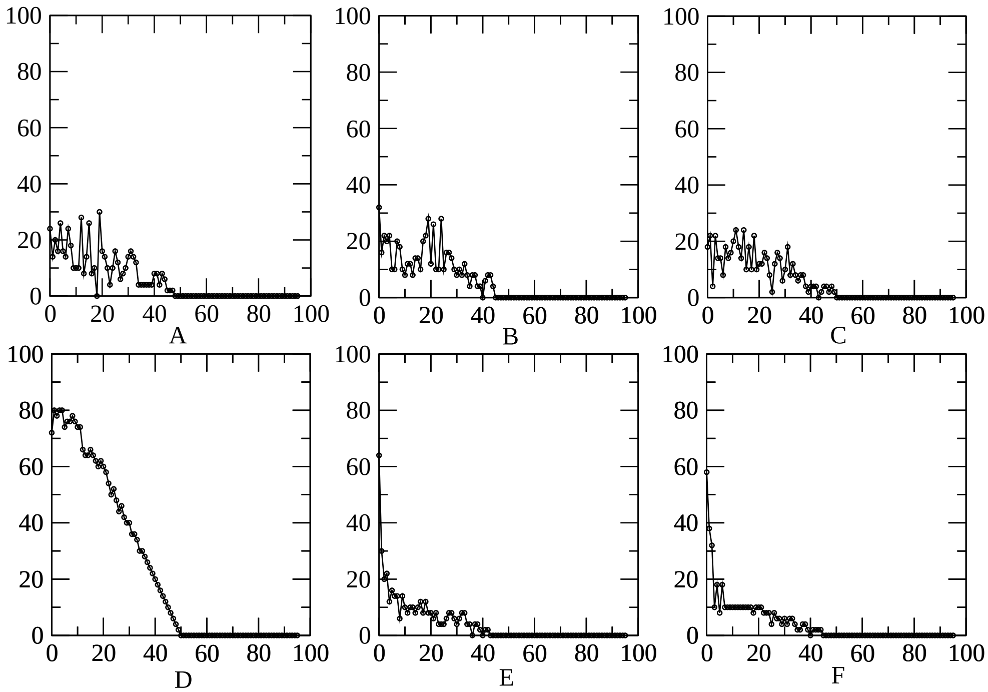


Figure 7. Integration and segregation. Left side, the Pareto front. Cross: configurations obtained through optimization. Square: configurations obtained by applying the rules of the self-organized dynamics to a random pattern. X axis: average utility of housing (negative). Y axis: average utility of retail (negative). Right side: the variation of I^d, the index of dissimilarity, in relation to W_h, the weight applied to the housing. X axis: W_h, Y axis: I^d.

The first time observed dissimilarity between the optimal and the self-organized configuration, results in the applying of the method for the convergence. The outcome has been evaluated by considering the relation: number of pinning cells–distance between optimal and self-organized configuration (D_{pq}). As figure 8 shows, the decrease of the distance is proportional to the number of pinning cells unless the last two cases (E and F, figure 8). But in all the cases the distance is zero only when the number of pinning cells equals 50, which coincides with the number of housing and retail land uses.



	A	B	C	D	E	F
W_h	0.05	0.2	0.4	0.6	0.8	0.95
W_r	0.95	0.8	0.6	0.4	0.2	0.05

Figure 8. Integration and segregation. The varying of distance D_{pq} with the increase of the number of the pinning cells in which the land use has been fixed. X axis: the number of the pinning cells, Y axis the distance D_{pq}.

Integration and segregation, second experiment

Because these results are influenced by the equal number of housing and retail land uses, a similar experiment has been performed with a different number of land uses, and with the inclusion of the open land. The number of land uses has changed as it follows: 50 cells for housing, 10 for retail and 40 for open space, while the matrix $\mathbf{M}$ has been slightly modified, as in table 3. Through this variation the degrees of freedom

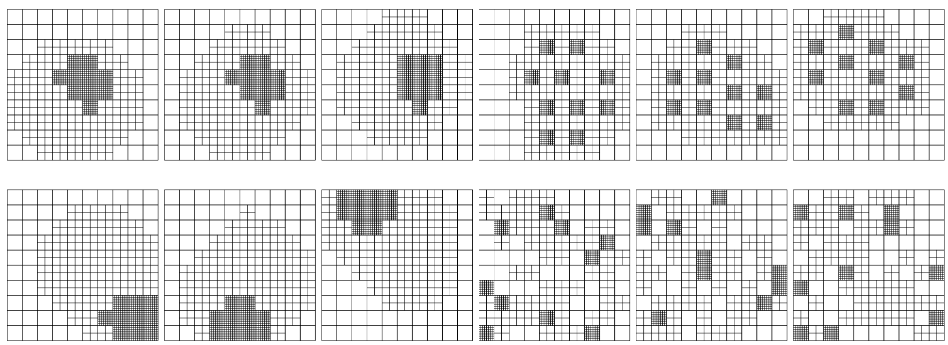
Table 3. Integration and segregation, second experiment. Matrix $\mathbf{M}$.

Integration and segregation		
Land use	Hous.	Ret.
Housing	0.1	0.9
Retail	0	1

of the housing and retail land uses, in the occupation of the surface increase. In fact, as figure 9 shows, the resulting patterns are quite different. The optimal configurations are twofold: the concentration of the retail land use in the center, which is similar to the result of the self-organized dynamics and an homogeneous distribution of retail regularly mixed with the housing land use which differs from the result of the self-organized dynamics. Thereby the Pareto front is reduced to only two points (figure 10). The index of dissimilarity I^d increases with the increase of the weight W_h assigned to the housing land use (figure 10).

In turn a low number of pinning cells is enough to obtain an important decrease of the distance (figure 11). In fact in the first ten pinning cells (see graphs D, E, and F of figure 11) are just located the ten retail land uses, which influence the location of the housing cells.

This last aspect highlights the relation between the self-organized dynamics and the optimization process. These two processes converge when the maximum weight is assigned to a land use which plays a *central* role in the the interaction. The centrality of a land use can be roughly calculated by using the difference: sum by columns minus sum by row in the matrix $\mathbf{M}$ (table 3) i.e. the capability to influence minus the degree of being influenced, as it has been done with the calculation of the influence index. It is easy to conclude that in this experiment the retail land use plays the most central role. Hence the optimal configuration and the outcome of the self-organized dynamics coincide when a big weight is assigned to the retail land use. In turn, a lot of energy has to be spent to compel the system to converge with the optimal configuration, when the maximum weight is assigned to a land use which does not play a central role, in the sense previously defined.



W_h:	0.05	0.20	0.40	0.60	0.80	0.95
W_r:	0.95	0.80	0.60	0.40	0.20	0.05

Housing Retail Open land

Figure 9. Integration and segregation, second experiment. First row. The patterns obtained through optimization. Second row. The patterns obtained by applying the rules of the self-organized dynamics to a random pattern. At the bottom of each column the set of weights utilized for housing (W_h), and retail (W_r) is shown.

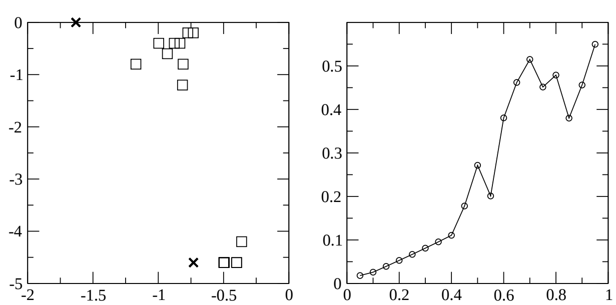
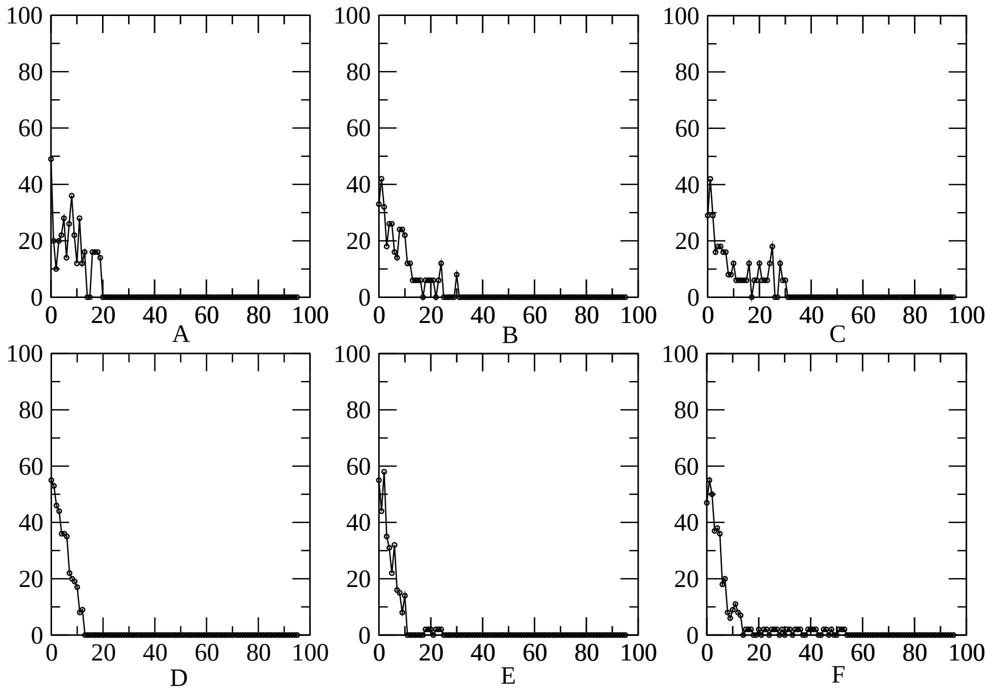


Figure 10. Integration and segregation, second experiment. Left side, the Pareto front. Cross: configurations obtained through optimization. Square: configurations obtained by applying the rules of the self-organized dynamics to a random pattern. X axis: average utility of housing (negative). Y axis: average utility of retail (negative). Right side: the variation of I^d, the index of dissimilarity, in relation to W_h, the weight applied to the housing. X axis: W_h, Y axis: I^d.



	A	B	C	D	E	F
W_h	0.05	0.2	0.4	0.6	0.8	0.95
W_r	0.95	0.8	0.6	0.4	0.2	0.05

Figure 11. Integration and segregation, second experiment. The varying of distance D_{pq} with the increase of the number of the pinning cells in which the land use has been fixed. X axis: the number of the pinning cells, Y axis the distance D_{pq}.

A more realistic experiment

The following experiment is conceived in a way that is less schematic and more similar to the reality. Two new land uses, industry and equipment, are added and each cell can be assigned to one of the following land-uses: housing (30 cells), retail (5 cells), industry (10 cells), open land (53 cells), and equipment (2 cells). The location of equipment is fixed, in order to simulate the existence of some exogenous factors. The matrix $\mathbf{M}$ is shown in table 4. As usual, positive effects occur for housing in case it has in the nearby

Table 4. Matrix $\mathbf{M}$, and number of land uses.

Land use	Housing	Retail	Industry	Open land	Equipment	Number of land uses
Housing	0.3	0.4	0.1	0.2	0.0	30
Retail	0.35	0.5	0.15	0.0	0.0	5
Industry	0.0	0.1	0.2	0.0	0.7	10
Open land	0.0	0.0	0.0	0.0	0.0	53
Equipment	0.0	0.0	0.0	0.0	0.0	2

cells retail and open land uses, and in a lower degree, industry in which workplaces are located. Positive effects occur also for the retail activities in relation to housing and industry which represent in various degree a demand for retail. Industry utility increases with the contiguity to the equipment.

Figure 12, shows the outcomes of the experiment including also the influence index of each cells, while in figure 13 the Pareto front is reported in three views. The convergence depends on the weigh assigned to the retail land use which at a first insight appears as the most central in the interaction rules. However, to calculate more rigorously the index of centrality of a land use we reason in the following way. Reminding that m_{hr} is the utility for use h to be surrounded by use r, m_{rh} is a measure of the influence of r on h. The more the quantity of land uses h, the more the absolute influence of r. For this reason a matrix $\mathbf{M}'$ is defined which is the transpose of the matrix $\mathbf{M}$, where each element is multiplied by the probability to find the influenced land use use:

$$m'_{rh} = m_{hr}\frac{N_h}{n^2}. \tag{16}$$

Similarly as for matrix $\mathbf{G}$, the direct and indirect influence is calculated as in the following expression:

$$\mathbf{M}^\star = \mathbf{M}' + \mathbf{M}'^2 + \mathbf{M}'^3 + \dots \tag{17}$$

The index of the centrality for the land use h (I_h^c) is given by the sum by row minus the sum by columns, in other words the capacity to influence minus the degree of being influenced:

$$I_h^c = \sum_r m^\star_{hr} - \sum_h m^\star_{hr}. \tag{18}$$

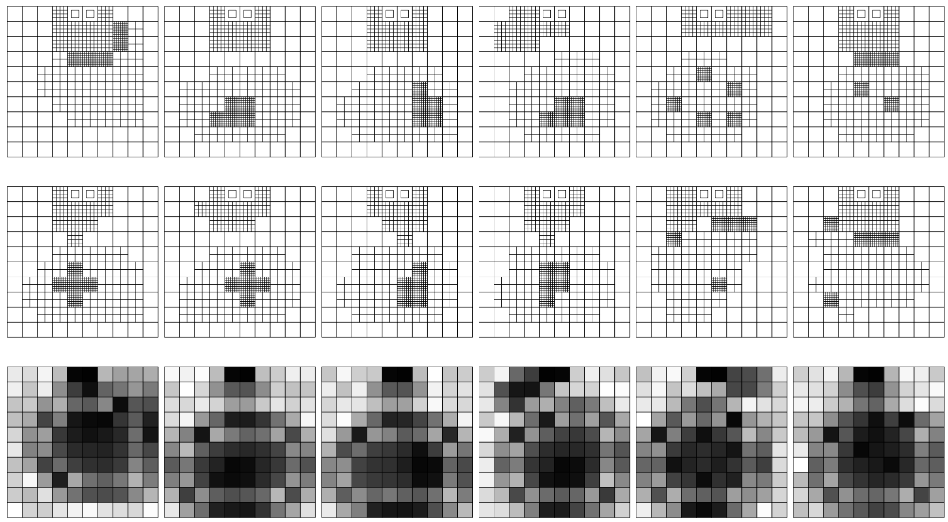
After having established a method for calculating an index of centrality we can forecast the relation between the optimal configuration and the self-organized dynamics. According to the result, shown in the table 5, last column, all the efficient solutions generated with a big weight assigned to the retail land use should coincide with the result of the self-organized dynamics. In fact, as figure 12 shows, the optimal configuration matches the self-organized dynamics when a big weight is assigned to the retail land use. In addition, the consequences of the establishment of pinning cells are shown in figure 14. The distance decreases quickly with the increase of the number of the pinning cells when the weight of the land uses and the centrality index does match (see the cases B and C, figure 14). In case they don't, the central role of the retail land use is utilized to control the other land uses (cases A and F, figure 14, in which the first chosen pinning cells are mostly retail land use). In essence in this case the control of an only limited part of the pinning cells produces a convergence, with a limited amount of energy spent in the control.

Table 5. Elements of the matrix $\mathbf{M}^\star$.

Land use	Housing	Retail	Industry	Open land	Equipment	Centrality index (I^c)
Housing	0.09899	0.01983	0.00020	0	0	-0.22033
Retail	0.13632	0.02316	0.01053	0	0	0.11671
Industry	0.03497	0.00851	0.01861	0	0	-0.03929
Open land	0.06661	0.00120	0.00001	0	0	0.06782
Equipment	0.00247	0.00060	0.07202	0	0	0.07510

A 30×30 grid experiment

Finally and in order to discuss the scalability of the proposed method the spatial grid has been enlarged to 30×30 squared cells. The rules of interactions included in matrix M are the same unless the neighborhood Φ_i utilized to calculate the spatial relations which has been enlarged to the 48 cells included in the square of 7×7 cells around the central cell in question. This enlargement is necessary in order to simulate the long range spatial relations, and it is the only change which assures the scalability of the proposed method.



W_h:	0.05	0.05	0.05	0.50	0.90	0.50
W_r:	0.05	0.50	0.90	0.45	0.05	0.05
W_i:	0.90	0.45	0.05	0.05	0.05	0.45

Housing Retail Industry Open land Equipment

Figure 12. The more realistic experiment. First row. The patterns obtained through optimization. Second row. The patterns obtained by applying the rules of the self-organized dynamics to a random pattern. Third row: the rank of pinning cells in relation to the influence index (I^{i}). The gray scale represents the influence of the cells: the most influent cells are black, the less are white colored. At the bottom of each column the set of weights utilized for housing (W_h), retail (W_r) and industry (W_i) is shown.

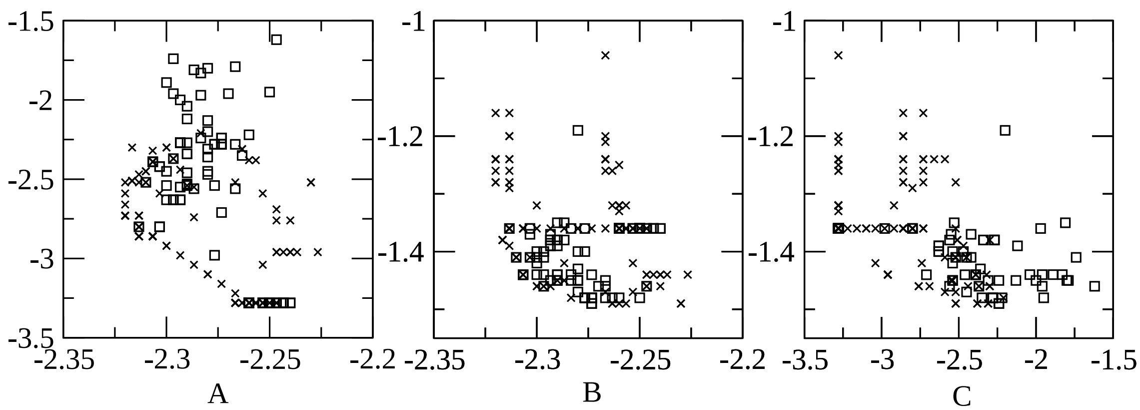
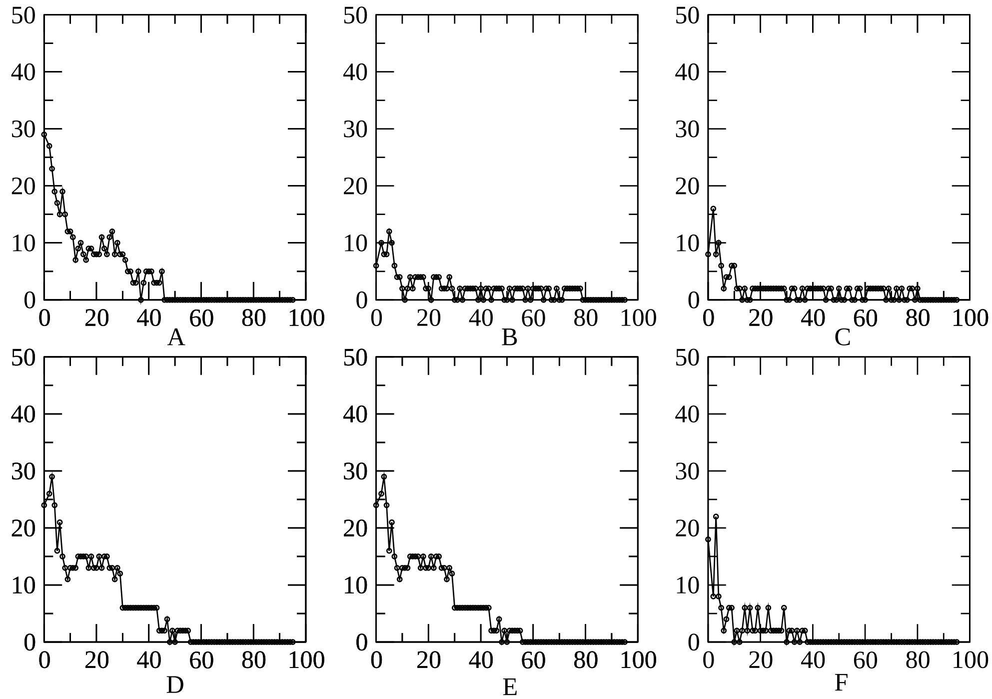


Figure 13. The more realistic experiment. The Pareto front. Cross: configurations obtained through optimization. Square: configurations obtained by applying the rules of the self-organized dynamics to a random pattern. A: X axis: average utility of housing (negative). Y axis: average utility of retail (negative). B: X axis: average utility of housing (negative). Y axis: average utility of industry (negative). C: X axis: average utility of retail (negative). Y axis: average utility of industry (negative).

The number of land uses has been proportionally increased: housing: 270 cells, retail: 50 cells, industry: 90 cells, open land: 470 cells and equipment: 20 cells. Only two cases are shown (figure 15) which utilize the same set of weight as the first and the sixth cases shown in figure 12. The analysis of the convergence



	A	B	C	D	E	F
W_h	0.05	0.05	0.05	0.50	0.90	0.50
W_r	0.05	0.50	0.90	0.45	0.05	0.05
W_i	0.90	0.45	0.05	0.05	0.05	0.45

Figure 14. The more realistic experiment. The varying of distance D_{pq} with the increase of the number of the pinning cells in which the land use has been fixed. X axis: the number of the pinning cells, Y axis the distance D_{pq}.

in relation to the number of the pinning cells is shown in figure 16. The results are similar to the 10×10 experiments, even if a finer tuning of the interaction matrix is requested in order to obtain a pattern less schematic and more similar to that observed in the reality.

Relation with the urban planning and control

In this experimental section we have found that the optimization process and of the self-organized dynamics produces similar results when the weigh assigned to the land uses coincide with the index of centrality of land use. However when this coincidence does not happen, the land use with an big index of centrality can be easily utilized to control the self-organized dynamics. These conclusions have to be related to some experience of urban planning especially in the fields of the regeneration of existing areas[17]. In this case what planners are looking for is the catalyst effects of the urban project[21]. This is very similar to obtain the desired plan with the control of a limited number of cells. The catalyst is in fact like a pinning cell in which planning effort are concentrated in order to stimulate the development in the desired direction. However the identification of the critical point [22] where to concentrate investments is a further possible utilization of the proposed method. In this way the strict zoning control could be relaxed in order to allow the network of local actions to operate more freely.

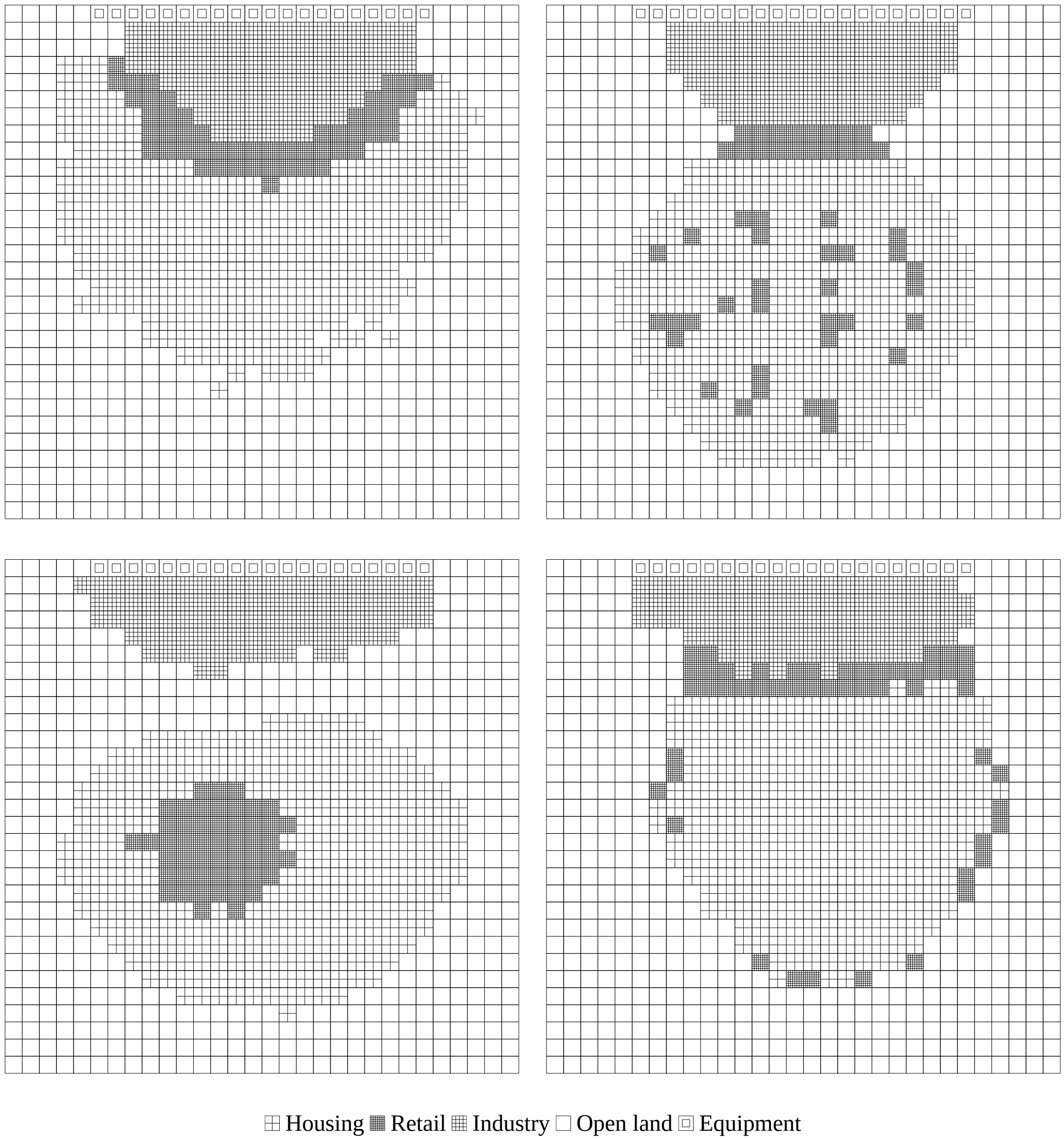


Figure 15. The 30×30 grid experiment. First row. The patterns obtained through optimization. Second row. The patterns obtained by applying the rules of the self-organized dynamics to a random pattern. The two columns of graphs refer to the following set of weights: first column: $W_h = 0.05$, $W_r = 0.05$, and $W_h = 0.9$; second column: $W_h = 0.5$, $W_r = 0.05$, and $W_h = 0.45$. They have to be compared with first and sixth cases in figure 12.

Conclusion

We have shown that optimization and self-organized dynamics can be conceived as similar process. From one side the optimization is a special kind of dynamics, and from the other side the self-organized dynamics is a special kind of optimization. In other words the urban systems are also problem solving. In turn the solving of the problem can be conceived as a dynamic process in which at each step the utility arising from each couple of bordering cells is considered instead of focusing in the utility of only one land use, and

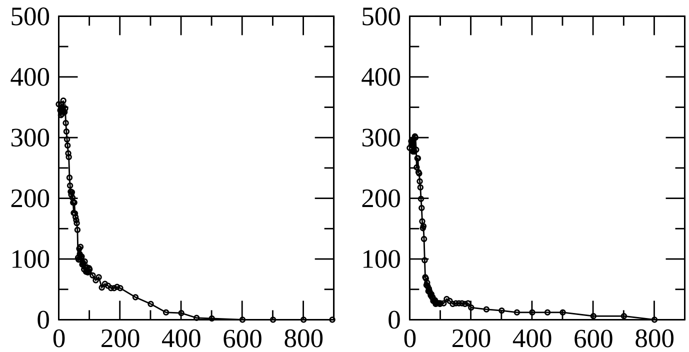


Figure 16. The 30×30 grid experiment. The varying of distance D_{pq} with the increase of the number of the pinning cells in which the land use has been fixed. X axis: the number of the pinning cells, Y axis the distance D_{pq}.

with the utilization of an exogenous factor, i.e. the cooling, which allows the system to reach and maintain the optimal configuration. Sometimes the attractors of the self-organized dynamics and of the optimization process are similar. This is the Utopia in which individual and total utility do coincide. In case this does not happen, as usually, instead of using the control extended to each point of the surface that totally constraints the self-organized dynamics with the plan, we have proposed a method which takes advantage of the self-organizing character of the urban dynamics and minimizes the controlling effort. Finally, this method can be applied to the urban planning practice based on the research of the element able to produce positive catalytic effect on the whole urban structure.

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