Assigning Measures of Complexity to Subsystems

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Introduction

Entropy and other measures of complexity are shown to be properties which may be assigned to subsystems [1]. These quantities are relative measures which are the result of the observer's position and perspective. The observer's perspective is defined by his internal organisation and his assignment conditions, which state what part of the system belongs to the observer and what part belongs to the world [2]. This observer may be a human brain but is also conceivable as a nested detector with anticipatory faculties.
In a phenomenological approach, macroscopic manifestations of underlying correction processes (resulting, e.g., in temporal or auditory illusions), which are generated by the observer, are identified. It is proposed that mechanisms such as these correction processes form a fractal interface between the observer and the world. This fractal interface, again, generates the observer's perspective of the world. Against the background of the Theory of Fractal Time [3], a differentiation between observer types or nested detectors in terms of their internal differentiation is proposed [4].
Measuring results for entropy and other measures of complexity will differ with the individual observer types or nested detectors, as the relative quantities of the former are determined by the fractal interfaces of the latter [5]. The notion of interface complexity is introduced, which takes account of the macroscopic manifestations of the observer's assignment conditions.

Keywords: fractal time, fractal interface, assignment conditions, interface complexity

Open, Closed and Nested Systems: a *Gedankenexperiment* (Thought Experiment)

There is a persisting notion that there is an increase of entropy in our universe. Supporters of this theory will agree that, locally, entropy may decrease: in a refrigerator, for example, energy is pumped out of the system, which leads to a low state of entropy. This is only possible, however, they may argue, because a powerplant, which produces the energy consumed by the fridge, generates an increase in entropy. Taken as one system, the subsystems of fridge and powerplant generate an increase in entropy. This statement would raise no objections.
When we start talking about the entire universe, however, this notion is not easily tenable. For a start, we do not know the extensions of our universe, nor what it actually consists of nor whether it is finite. Is an infinite universe an open or a closed system? Against the background of such questions, it is rather brave to claim that, in total, entropy increases. This objection may be illustrated by a thought experiment based on nested levels of description.
If we imagine a universe consisting entirely of nested ice cubes and hot water bottles (see Figure 1), the difference in entropy ($\Delta S = S2-S1$) would increase in the ice cubes (nested in hot water bottles) and decrease in the hot water bottles (nested in ice cubes) [1]. As we do not know whether the outermost embedding structure of the universe is an ice cube or a hot water bottle, we cannot say whether entropy increases if we consider the entire universe as a reference frame.

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Another problem arises when we want to measure entropy in the ice-cube-and-hot-water-bottle-universe. In order to measure the difference in entropy between two (freeze-frame) states, an observer or a detector would need to be positioned somewhere in this universe. As this observer also consists of matter, takes up a certain amount of space and displays an internal differentiation, the observer's position and internal differentiation have to be taken into account when we want to measure ΔS. Only a super-observer with an exo-perspective [6], who is positioned outside this universe, could describe a measure of complexity such as entropy, without being a part of the system he wishes to describe. Scientists are, in general, part of the universe they set out to describe. Therefore, they are subjected to an endo-perspective [6] – they are part of the system they want to describe.

Figure 1

If the observer is positioned in a subsystem of the ice-cube-hot-water-bottle-universe, and if his internal differentiation resembled that of the ice-cube-hot-water-bottle-universe, would he have to be regarded as an open or as a closed system [1]?
Let us assume the observer himself consists of nested subsets with increasing and decreasing entropy, mirroring the structure of the universe he is embedded in. If he then monitors the universe he is part of, he would not detect any increase or decrease in entropy for the subsystem he "covers", as the structural congruence between the observer and the system to be observed would lead, as one-to-one mapping, to an observation of no change in entropy. This idea was expressed in a nutshell by O.E. Rössler: "It could turn out, for example, that a universe that is chaotic itself *ceases* to be chaotic as soon as it is observed by an observer who is chaotic himself." [6]

This also applies for other measures of complexity in which a relative, interfacial complexity measure shaped by the observer-world relation must be assumed: for any measure taken by an observer who is part of the system he wishes to observe.

The observer's position and perspective, which results from his internal differentiation, determine the degree of complexity measured by this observer.

The Observer's Internal Differentiation and Assignment Conditions

When we describe observer-world interaction, we have to take account of assignment conditions, which state what part of a system belongs to the observer and what part belongs to the world. The notion of assignment conditions goes back to Rössler [7], who had microscopic assignment conditions in mind when he developed the idea. Here, however, I shall refer to the *macroscopic manifestations* of assignment conditions.

In the case of the perception of an auditory illusion, the illusion is a qualium which may be attributed to the manifestations of the observer's assignment conditions, whereas the air pressure waves may be attributed to the manifestations of the world's assignment conditions.

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This interfacial cut may be set between the observer's brain and a measuring chain or between the observer and the outside world. This observer could also be a smart detector with anticipatory faculties, i.e., a detector which can modify the structure of its own interface. How could one detect and measure the observer's internal differentiation? One methodological candidate is to describe it in terms of correction processes.

Correction Processes

Complexity may be defined as a measure which is determined by the structure of the observer's Now – his present and only window to the world. The structure of the observer's Now is shaped by (among other pattern-forming constraints) correction processes. For my purposes, correction processes which underlie visual and auditory illusions are a convenient starting point. To begin with, I shall briefly introduce a correction process underlying a visual illusion and draw an analogy with a correction process underlying an auditory illusion.

We tend to see familiar objects as having standard shape, size, colour or location, even when the perspective, distance or lighting changes (e.g. a car approaching us fast while we are walking towards it on the roadside). Our impression tends to conform to the object as it is or is assumed to be, rather than to the actual stimulus. This perceptional constancy makes it possible for us to identify objects under varying conditions. Apparently, we take these conditions into account when we process and interpret our perceptions.
This stability in perception seems to be persistent despite the fact that there is considerable instability in the stimulation. We take distance and relative size into account when we observe an object in its surroundings: For example, we see objects as of the same size at different distances because they stay the same size relative to surrounding objects. We are usually not aware of this internal correction process. Unless, of course, objects of the same size are not of the same size relative to surrounding objects - then we experience the so-called *corridor illusion* (see Figure 2).

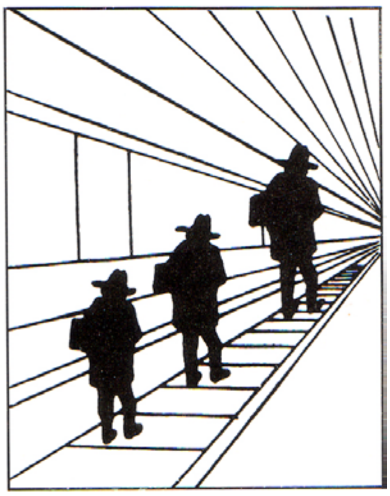

Figure 2 [8]

A case-differentiation for different types of observers is necessary here: It is important to remember that the truth value of the observer's communicated perception of the corridor illusion depends on the levels of description (LODs) available to this observer: It depends on whether the observer interprets the 2-dimensional visual representation as a 2-dimensional or a 3-dimensional object.

The corridor illusion only occurs if the 2-dimensional object is interpreted as a 3-dimensional one, i.e., against the background of an additional LOD which the observer has generated. An observer who does not have the ability to fall back on such an embedding LOD, sees the three men as being of the same size, without reference to the corridor perspective.

Visual and auditory illusions with underlying correction processes facilitate compatibility with the outside world. As "successful" observers, we interpret the 2-dimensional representation as a 3-dimensional one and thus experience a (visual) illusion: We perceive the three men in the corridor as being of different sizes as a result of their relative positions to the background, which we take into account. If we did not add this (physically non-existent)

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dimension, we would not encounter the illusion. This acquired perspective maybe misleading in the "correct" estimation of the size of the three men in the corridor illusion. In our everyday lives, however, our 3-dimensional perspective allows us to successfully interpret and navigate through the world.

Auditory illusions such as the Shepard scale are also based on underlying correction processes: "A *Shepard tone* is a sound consisting of a superposition of tones separated by octaves. When played with the base pitch of the tone moving upward or downward, it is referred to as the *Shepard scale*. This creates the auditory illusion of a tone that continually ascends or descends in pitch. (...) This can be constructed by creating a series of overlapping ascending or descending scales (...) Overlapping notes that play at the same time should be exactly an octave apart, and each scale should fade in and fade out, so that it is impossible to hear the beginning or end of any given scale." [11] The discrete Shepard scale (Figure 3) displays a self-similarity in its signal, which prompts an auditory illusion because the listener focuses only on pitch relations and thereby tries to extract a one-dimensional signal from a multi-layered one. The colours indicate the volumes of the notes with purple being the quietest. [11]

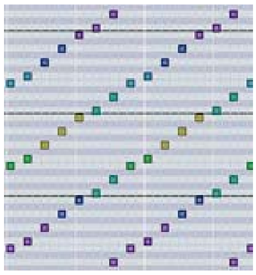

Figure 3

If the signal is multi-layered to start with, a multi-layered interface will simplify the signal for the listener. By differentiating the layers, the listener will not perceive an auditory illusion. He will be able to hear distinguishable parallel tone sequences. [12] If we may thus assume that correction processes underlying auditory illusions appear to change the structure of the observer's Now, his interface with the world, the next question to consider is: How does an observer's Now need to be structured in order to perceive an auditory illusion?

The Observer's Extended Now

In this paper, it is assumed that the Now is not a point but has extension and a fractal structure generated by the observer and the world around him. This assumption is based on the following considerations: The German philosopher Edmund Husserl first described a nested structure of the Now [13]. He pointed out that when we listen to a tune, we hear a succession of musical notes. But we do not perceive simply a succession of unrelated notes - we hear a tune. We are able to do this because we internally connect the note we have just heard with the present one and the tone we anticipate to follow it. But we do not connect them in an arbitrary way: we remember a tone (*retension*) and anticipate the next tone (*protension*) within the consciousness of the present, the Now. As we do this over and over, we create a nested temporal pattern within the Now (see Figure 4). Without memory of the preceding note and no anticipation of the next one, we would only perceive a succession of isolated, unrelated

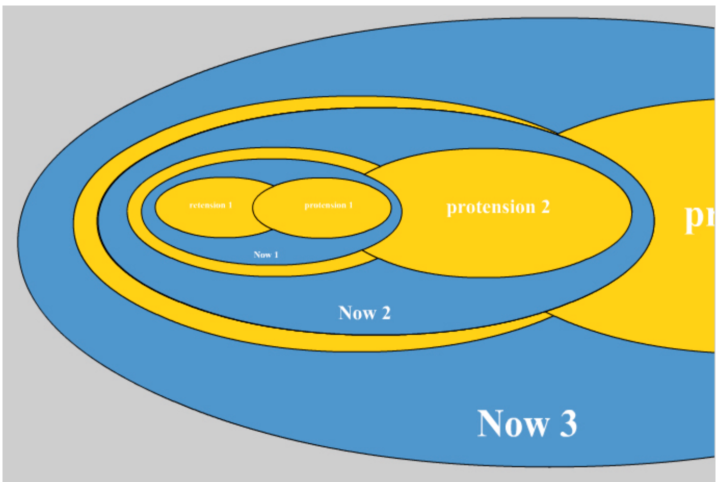

Figure 4

notes. But as we are able to perceive a tune as opposed to a succession of isolated notes, we must assume the Now to host *both* succession and simultaneity. Succession and simultaneity within the Now generate a nested, fractal structure. In order to explain our ability to perceive a tune or any other time series as a meaningful entity, we must assume the observer's Now to have extension and a nested structure.

Fractal Time

My Theory of Fractal Time [3], takes account of the observer's nested Now by differentiating between Δt_{length}, Δt_{depth}, and $\Delta t_{density}$. Δt_{length}, the length of time, is the number of incompatible temporal extensions in a time series. It measures the succession of events on one LOD. Δt_{depth}, the depth of time, is the number of compatible temporal extensions in a time series. It measures simultaneity and provides the framework time which allows us to structure events in Δt_{length}. $\Delta t_{density}$, the fractal dimension of time, is the temporal density of a time series. It is important to realize that Δt_{depth} logically precedes Δt_{length}: There is no Δt_{length} without Δt_{depth}, i.e., no succession is conceivable without a presupposed embedding simultaneity.

Fractal and Non-Fractal Observers

There are two types of observers: fractal and non-fractal [9], [10]. A non-fractal observer has no nesting faculties and can therefore not generate the embedding level of description which would allow him to hear a tune. This observer type can perceive only isolated notes in a tune or isolated events in a time series. As he would not be able to generate a temporal fractal perspective through continuous nestings, simultaneity, succession and memory formation would be unknown to him. Thus, no learning or reflection could take place. The non-fractal observer would live in an eternal succession of unconnected Nows.
A fractal observer, on the other hand, is able to hear a tune, as he can observe events on a number of LODs. This enables him to generate a nesting cascade of LODs, a temporal fractal perspective, which allows him to observe succession and simultaneity of events directly, in real time. [9] A temporal fractal structure which appears to be a highly complex time series (measured with conventional methods) would appear as not very complex to an observer with a temporal fractal interface containing the same internal structure (number of nestings (LODs) and scaling factors).

Observer-Frame Complexity and Interface Complexity

Two measures of complexity which take account of assignment conditions are conceivable [12]: The first, observer-frame complexity, measures the internal differentiation of the observer in terms of Δt_{depth}. Such a differentiation, which shapes the structure of the observer's interface, may be revealed by registering internal correction processes, e.g., the perception of auditory illusions by an observer (listener). It is measured in the number of (simultaneous) LODs. The second, interface complexity, measures the number of simultaneous "disentangling performances" carried out by the observer (defining the degree of complexity reduction). If he were, for example, observing a section of the ice-cube-and-hot-water-bottle universe he is part of, and whose structure matches his internal differentiation, the observer would measure neither an increase nor a decrease in entropy.

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To conclude, complexity is a relative, LOD-dependent measure. The measure of interface complexity will decrease with the number of LODs available to the observer and increase, if there is little or no one-to-one mapping between the observer's internal structure and the structure of the embedding outside world.

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