

Hierarchical Organization in Smooth Dynamical Systems

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August 31, 2005

Abstract

We define and characterize hierarchical structures in smooth dynamical systems. Smooth projective maps from the original phase space to a phase space with lower dimensionality define transitions between levels in a dynamical hierarchy. It is required that each level describes a self-contained deterministic dynamical system. We show that a necessary and sufficient condition for a projective map to be a transition between levels in the hierarchy is that the kernel of the differential of the map is tangent to an invariant manifold with respect to the flow. The implications of this condition are discussed in detail. We also show how the projections can be defined using a quotient manifold construction with a Lie group invariant under the flow of the dynamical system. The relation to symmetries of the dynamics is demonstrated. Finally these results are used to define functional components on different levels, interaction networks, and dynamical hierarchies.

Acknowledgment: This work was funded by PACE (Programmable Artificial CellEvolution), a European Integrated Project in the EU FP6-IST-FET Complex Systems Initiative, and by EMBIO (Emergent Organisation in Complex Biomolecular Systems), a European Project in the EU FP6 NEST Initiative.