

Chaos and budworm dynamics in Multi-Agent Systems (MAS): A bio-inspired approach to the spreading of novelty

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Catastrophic events in MAS

A large number of real systems exhibit dynamics that bear the potential for chaos. The domains over which stability of the systems occurs can be relatively large. But once in a while, systems may move towards the edge of stability and little nudges to the system may move it from stability to instability, that is, into a catastrophe. Subsequently, reorganisation of system components, which may occur as a reaction to bring the system back into a stable domain, a kind of evolutionary process. This stable domain, however, may not be the same as the one prior to the disturbance. In this new stable domain a novelty spreads by meanings, objects and subjects, and produces innovation.

The system undergoes a catastrophic event in the sense that it is moved from an initial state of stability through a dramatic phase of reorganisation and back to some degree of stability. Examples of such catastrophic events include landslides, avalanches, earthquakes, and pest outbreaks in ecosystems. In each case, small changes in the system occur, where these individually may not be critical to the system's behaviour. Collectively, however, they lead to the evolution of the system towards a critical state. This is apparent, for example, in the case of pest outbreaks in ecosystems. Each infected individual potentially adds to the instability of the system. When a critical point is reached, the next infected individual may trigger an epidemic that affects a larger part of the system. Temporary stability is quickly reached if the epidemic is not too intense. Even if not in a large scale, the epidemic adds to the stress of the system's fragile regions, making these more susceptible to further epidemics as more individuals are infected at those regions or as additional diseases are received from the most fragile regions of the system. Ultimately, a large-scale catastrophic event may occur that affects the entire system, not just individual regions. The system components regroup and finally enter a phase of new, temporary stability.

Evolutionary processes are hence at work, making the system more efficient. We refer to this as evolution toward catastrophe. A system in such a state can transgress to a stable state by another process of evolution, faster than the first kind, where this new stable state may not be very efficient. Large living natural systems are similarly constrained from operating at or near peak efficiency by random intervention of uncoordinated external processes at the regional levels.

We can develop a simple model of catastrophe and then proceed to create an epidemic model. Before this, we consider the following equation (Beltrami, 1998):

$$x^3 - \alpha x - \beta = 0 \quad (1)$$

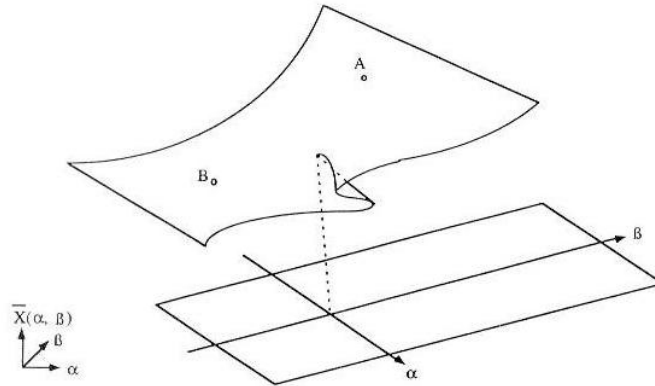


Figure 1.

Consider Figure 1, which illustrates the surface defined by Eq.(1). Imagine a ball lying at the top of this surface such as point A. The ball may be still, and very small nudges away from its first equilibrium point A, leading to a new equilibrium. After a series of such small perturbations, however, the ball will roll off the top part of the surface, and a priori it is difficult for us to determine exactly where it will end up. All we know for certain is that the new equilibrium position is somewhere at the bottom of the surface, say point B. Small nudges to the ball in B will again move it slightly away from B. And if we push it hard enough, we can propel the ball through the fold, or cusp, to the upper part of the surface again. Where exactly will it end up? A precise answer requires exact knowledge of the shape of the surface, the properties of the ball, and the magnitude and direction of the force exerted on the ball. In more complicated real-life systems not all the variables describing the system and the forces exerted are known well enough. As a result, we may only know stability domains rather than specific locations.

Budworm Dynamics

A classic example for the implications of ecosystem catastrophes is budworm dynamics (Royama, 1984). Budworms are caterpillars that feed on spruce and fir forest in north-eastern Canada and the United States. When forest stands reach maturity, budworm populations explode, seriously affecting the forest by defoliating the trees. With the death of trees comes a loss of food and consequent population crash. The cycle of [low population density]-[stand maturity]-[budworm population explosion (and collapse)] tends to repeat itself over a lapse of time. A natural systems way of controlling, amongst others, relates to the patch size. Natural systems no doubt avoid large catastrophes because they operate in patches, where the degree of maturity of adjacent tree patches is nearly always different. Consequently, pests and disease find difficulty in spreading beyond a patch, and the size of the catastrophe is kept small. Current MAS practice seems to ignore such system behaviour. Our model is not a regional one and such inter-

patch dynamics are not captured. However, we have conceptualised such patch dynamics to combine with this conceptual-theoretical model.

To model a bio-inspired MAS catastrophe, we use a budworm dynamics model (see Beltrami, 1998). Let us denote B as the agent population density, k the information carrying capacity, S the network density, and r_b the agent population growth rate. Thus,

$$\frac{dB}{dt} = r_b B \left(1 - \frac{B}{kS} \right) \quad (2)$$

describes the population dynamics for a fixed information carrying capacity and no negative influences on agent population growth. This is a logistic growth equation. Let us introduce the effects of a negative agent population growth factor, with a maximum rate of c , which is assumed to be constant. At small densities, c has only a minor effect on agent population because it is small, and thus the probability of those affected by the negative factor is also small. A negative agent population growth factor that captures such interactions is:

$$\frac{cB^2}{a^2 + B^2} \quad (3)$$

With a as a scalar that captures the effectiveness of the ‘negative agent population growth factor’ to affect and eradicate an agent. In a developed (more populated) system of agents, it is assumed that the negative factor is more effective than in a less developed system. Thus, a may be assumed to increase with increased development of the MAS, that is, network density

$$a = mS \quad (4)$$

And thus

$$\frac{cB^2}{a^2 + B^2} = \frac{cB^2}{(mS)^2 + B^2} \quad (5)$$

Where m is a constant.

Combining negative population growth with the logistic function yields

$$\Delta B = \frac{dB}{dt} = r_b B \left(1 - \frac{B}{kS} \right) - \frac{cB^2}{(mS)^2 + B^2} \quad (6)$$

Which is the equation used in the model that drives agent population changes, ΔB .

Changes in network density are assumed to also follow the logistic growth curve, with r_s as the rates of increase and k_s as information carrying capacity.

$$\Delta S = \frac{dS}{dt} = r_s S \left(1 - \left(\frac{S}{k_s E} \right) \right) \quad (7)$$

Where E is the percentage of nodes of the network. The denser the network, the higher E becomes. The percentage of nodes is assumed to decrease as the average agent density per network density B/S increases. To model the reduction of stress as agent population decreases, we multiply B/S by E^2 . The combined effect of logistic growth in network and agent induced nodes loss is:

$$\Delta E = \frac{dE}{dt} = r_E E (1 - E) - \frac{p B E^2}{S} \quad (8)$$

Where r_E is the rate of nodes increase and p a proportionality factor.

Let us consider the case of $B \neq 0$ and introduce the following notation:

$$R = \frac{r_B m S}{c} \quad (9)$$

$$Q = \frac{k}{m} \quad (10)$$

And rewrite

$$B = m S x \quad (11)$$

It can be shown that the nontrivial equilibrium of Eq. (6) fulfil

$$R \left(1 - \frac{x}{Q} \right) = f(x) = g(x) \quad (12)$$

With

$$g(x) = \frac{x}{1 + x^2} \quad (13)$$

The left side of Eq. (12) is a straight line $f(x)$ with slope $-R/Q$. Equilibrium occurs where this line intersects with $g(x)$. Both, S and R increase in Q . At first, there is a single equilibrium, corresponding to the situation shown in Figure 1.

References

- Beltrami, E. (1998). *Mathematics for Dynamic Modeling*. Second Edition. Academic Press: London.
- Royama, T. (1984) Population dynamics of the spruce budworm *Choristoneura fumiferana*. *Ecological Monographs* **54**(4): 429-462.