Clusters of computations for a linear transition system

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Let M be a state transition system (S, τ) where S is a state space, τ is a transition function. We study the case when $S = \mathbb{R}^n \times \mathbb{Z}^m \times \Omega$, where Ω is a finite set. That is every state of the studied system is characterized by a real-valued n-vector $\vec{x} \in \mathbb{R}^n$; an integer-valued m-vector $\vec{v} \in \mathbb{Z}^m$; and one variable of a finite range $w \in \Omega$. A *simple linear update* is defined by the following three rules: $\vec{x} := A\vec{x} + \vec{a}$, $\vec{v} := B\vec{v} + \vec{b}$, and $w := c$. Here, A is an $n \times n$ matrix with real coefficients, B is an $m \times m$ matrix with integer coefficients; $\vec{a}$, $\vec{b}$ and c are constants of the corresponding types. *Linear updates* are built from the simple updates by means of `do-in-parallel` blocks, conditionals and nondeterministic choice (see [1, 2]).

We use the idea of predicate abstraction to define the basic equivalence relation on the set of computations [3]. Consider a set of predicates $P_1, \ldots, P_r$ over the state space S. Then

$$s_1 \approx s_2 \text{ iff } (P_1(s_1) \equiv P_1(s_2)) \wedge \cdots \wedge (P_r(s_1) \equiv P_r(s_2)), \qquad s_1, s_2 \in S.$$

Consider now two computations $\mathbf{s}' = (s'_0, s'_1, \ldots, s'_l)$ and $\mathbf{s}'' = (s''_0, s''_1, \ldots, s''_l)$ of length l. We define them to be equal if for each step i the corresponding states are equivalent:

$$\mathbf{s}' \approx \mathbf{s}'' \text{ iff } \forall i \in \{0, 1, \ldots, l\}(s'_i \approx s''_i).$$

The maximal possible number of different equivalence classes is $(2^r)^l$, where r is the number of the predicates, l is the length of computations. This is not feasible even for small values of r and l.

In our work we consider other definitions of equivalence relations on the set of all computations, each of which is a weakening of the relation defined above. The main tasks here are the following. First, to find a reasonable definition of equivalence with rather small number of the equivalence classes (see [1]). Second, to get a representative for every equivalence class or to prove that the class is empty.

References

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